Water flow and hydraulic conductivity in a frozen unsaturated sand

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Introduction

Freezing

Food
maximize land use and agriculture in cold regions

Climate change
water flow, storage, evaporation from frozen ground

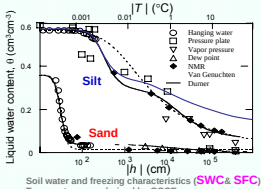
Artificial ground freezing
stabilize soil,form a barrier against hazardous waste

How soil water migrates during freezing?
How about the hydraulic conductivity?

Sample

Sand : Tottori dune sand **(Sand)**
mean di 0.35mm, pre-washed

Silt : Fujinomori clayey silt **(Silt)**
frost susceptible, contain 60% silt and 24% clay



Soil water and freezing characteristics (SWC& SFC).
Temperature was derived by GCCE.

$S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$

van Genuchten model

$S_e = \left(1+|\alpha h|^n\right)^{-m}$

Durner model

$S_e = w_1\left[1+(\alpha_1 h)^{n_1}\right]^{-m_1} + w_2\left[1+(\alpha_2 h)^{n_2}\right]^{-m_2}$

Freezing temperature
Surface forces make log-shape SFC

$T_m - T = \frac{AT_m}{6\sigma_{sl} L_f d^2}$

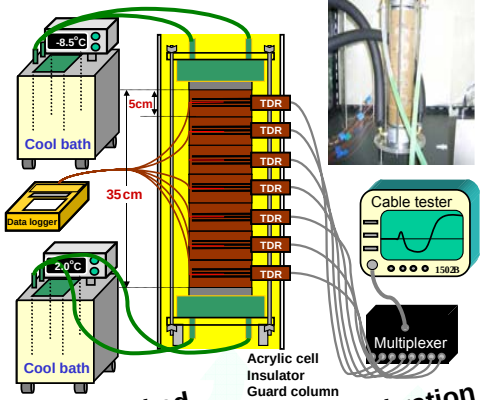
Pore distribution makes shoulder

$T_m - T = \frac{T_m \sigma}{\rho L_f r}$

		Sand	Silt
Bulk density	g cm⁻³	1.45	1.18
θ when packed	m³m⁻³	0.15	0.40
θ saturation	m³m⁻³	0.36	0.569
Thermal cond.*	Wm⁻¹K⁻¹		
at θ = 0.00 (frozen)		0.25(0.25)	0.20(0.20)
at θ = 0.17 (frozen)		0.96(1.50)	
at θ = 0.24 (frozen)		1.06(1.05)	0.52(0.55)
at θ = 0.29 (frozen)			0.66(0.76)
Saturated hydraulic cond.	cm h⁻¹	50.6	0.25
van Genuchten parameter			
θ_r	m³m⁻³	0.015	0.03
α	m⁻¹	0.336	0.0016
n		7.	1.38
l		−0.5	0.552
Durner parameter			
θ_r	m³m⁻³	0.	0.06
α_1	m⁻¹	0.03466	0.0035
n_1		6.4	3.1
l		−0.08	−0.5
α_2	m⁻¹	2.70x10⁻⁴	1.10x10⁻⁴
n_2		1.4	1.7
w_2		0.105	0.461

* The value for thermal conductivity is average of 2 to 20°C for unfrozen soil and −5 to−20°C for frozen soil.

Freezing experiment



TDR *vs* NMR

ε_r-θ_{ul} relationship can vary with ice content.

Modified Mixing Model

$$\theta(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha}) \quad (T \geq 0, \theta \geq \rho_b SD_c)$$

$$\theta_{ul}(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-\theta_{ul}(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha}) \quad (T < 0, \theta \geq \rho_b SD_c)$$

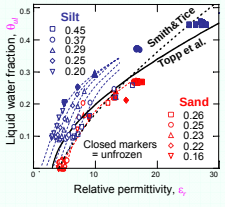
$$\theta = \sqrt{\rho_b SD_c \frac{(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-\theta_{ul}(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})}{\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha}}} \quad (T \geq 0, \theta < \rho_b SD_c)$$

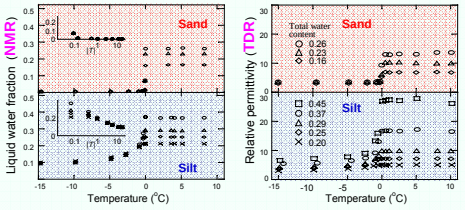
$$\theta_{ul} = \sqrt{\rho_b FSD_c \frac{(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-\theta_{ul}(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})-\theta_{ul}(\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha})}{\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha}}} \quad (T < 0, \theta < \rho_b SD_c)$$

where $\varepsilon_{sl,water}$ reduced by surface forces as

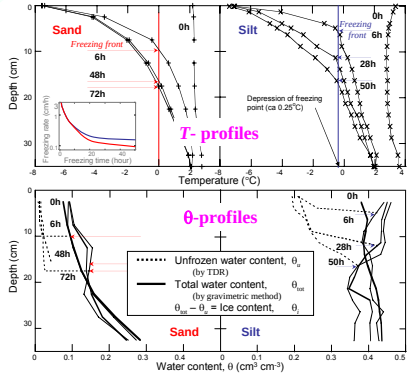
$$\begin{cases} \varepsilon_{sl}^{\alpha} = \frac{\varepsilon_{sl}^{\alpha}-\varepsilon_a^{\alpha}}{D_c \rho_b S}\theta + \varepsilon_a^{\alpha} & (\theta < D_c \rho_b S) \\ \varepsilon_{sl}^{\alpha} = \varepsilon_a^{\alpha} & (\theta \geq D_c \rho_b S) \end{cases}$$

and ice formation (F is a compression factor)

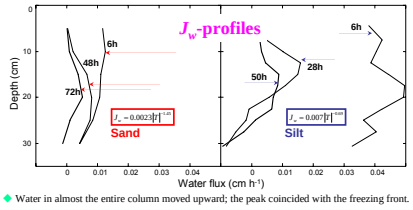




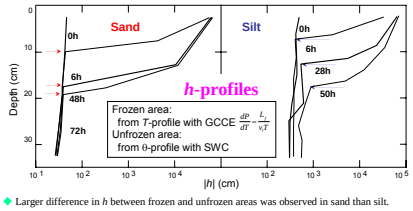
Results



- Sand froze slightly faster than silt because of the difference in the thermal conductivity.
- An increase of θ_{tot} decreased of θ_u in frozen area and decrease of θ in unfrozen area (water flow).
- In sand water accumulated at freezing front, while water flowed through frozen area in silt.

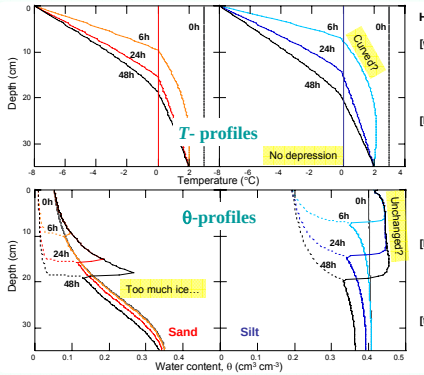


- Water in almost the entire column moved upward; the peak coincided with the freezing front.



- Larger difference in h between frozen and unfrozen areas was observed in sand than silt.

Numerical simulation



HYDRUS-1D code (modified for freezing)

[water flow]

$$\frac{\partial \theta_l(h)}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i(T)}{\partial t} = \frac{\partial}{\partial z}\left[K_{11}(h)\frac{\partial h}{\partial z} + K_{11}(h) + K_{12}(h)\frac{\partial T}{\partial z} + K_{vh}(\theta)\frac{\partial h}{\partial z} + K_{12}(\theta)\frac{\partial T}{\partial z}\right]$$

[heat flow]

$$\frac{\partial C_p T}{\partial t} - L_f \rho_i \frac{\partial \theta_i}{\partial t} + L_w(T)\frac{\partial \theta_l(T)}{\partial t} = \frac{\partial}{\partial z}\left[\lambda(\theta)\frac{\partial T}{\partial z}\right] - C_w\frac{\partial q_l T}{\partial z} - C_i\frac{\partial q_i T}{\partial z} - L_w(T)\frac{\partial q_l}{\partial z}$$

Flow equations are coupled through GCCE.
Durner model was used for SWC & SFC.

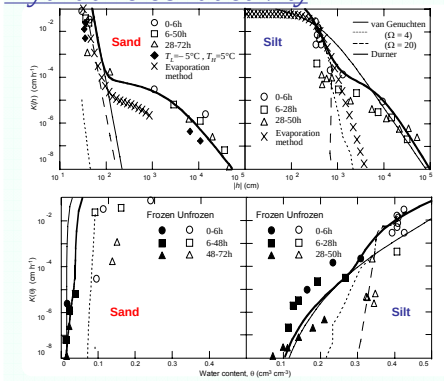
[hydraulic conductivity]

$$K(S_e) = K_s\left(w_1 S_e + w_2 S_e^{l}\right)^2 \times \frac{\left[w_1\alpha_1\left\{1-\left(1-S_e^{\frac{1}{m_1}}\right)^{m_1}\right\} + w_2\alpha_2\left\{1-\left(1-S_e^{\frac{1}{m_2}}\right)^{m_2}\right\}\right]^2}{\left(w_1\alpha_1 + w_2\alpha_2\right)^2}$$

[thermal conductivity]

$$\begin{cases} \lambda(\theta) = \lambda_0(\theta) + \beta C_w|q_w| \\ \lambda_0(\theta) = C_1 + C_2(\theta + F\theta_i) - (C_1 - C_4)\exp\left\{-\left[C_3(\theta + F\theta_i)\right]^{C_5}\right\} \end{cases}$$

Hydraulic conductivity



[water flow]

The simulation indicates that the vapor flow is very small.

$$\frac{\partial \theta(h)}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i(T)}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\frac{\partial h}{\partial z} + K(h) + K(h)\gamma\frac{\partial T}{\partial z}\right]$$

Thus

$$K(h) = \frac{J_w}{\left(\frac{\Delta T}{\Delta z}\gamma h + \frac{\Delta h}{\Delta z} + 1\right)}$$

- $K(h)$ changed at about h = -1000 cm.
  - decreased more gradually in frozen soil.
  - agreed with evaporation method.
- $K_l(h)$ in silt was about 10 times larger than in sand.
  - that is why more water flow in frozen silt.
- $K_l(h)$ also correlated with T and θ_i.
  - was well fitted by power law, as

Sand $K_l(h) = 0.7\times10^{-4}|h|^{-1.75}$; $= 8.2\times10^{-14}|\theta_i|^{2.42}$

Silt $K_l(h) = 3.0\times10^{-2}|h|^{-1.48}$; $= 2.3\times10^{-8}|\theta_i|^{4.56}$

- Use of the impedance factor requires caution.

$$K_l(h) = 10^{-\Omega\theta_i}K(h)$$

- Durner model well expressed the $K_l(h)$.

Summary

- The sand and silt columns were frozen directionally, and water and heat flow during soil freezing was measured. The flow depended on soil types. Numerical simulation agreed with the experiments.
- $K(h)$ for frozen and unfrozen soils was estimated by Darcy's law under non-isothermal conditions with ice formation. $K(h)$ steeply decreased with decreasing h and θ in unfrozen soil but more gradually decreased in frozen soil.
- Use of an impedance factor for calibrating $K_l(h)$ appears to be unnecessary when accurate SWC & SFC are available. Durner model was useful for expressing the hydraulic properties for frozen soils.

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