Simple Analytical Method to Estimate Soil Thermal Diffusivity in a Cylinder

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Background

1. Heat probe methods require specifically designed probes.
2. Soil cylinder methods need a numerical analysis.

Simple and inexpensive procedures have been sought by farmers.

Theory

Heat flow in soil in an axial symmetric coordinate is expressed as:

$$\frac{\partial T}{\partial t} = \kappa\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad \text{....................[1]}$$

where T is the temperature (°C), κ is the thermal diffusivity (m^2 s^{-1}), r is the distance from the center (m), and t is time (s).

For

Initial condition: T_i (°C) then T_r=0

Boundary condition: T_o (°C) then T_r=1,

Carslaw and Jaeger (1959) derives the following analytical solution:

$$T_r = \frac{T - T_i}{T_o - T_i} = 1 - 2\sum_{n=1}^{\infty}\exp\left(-\beta_n^2\frac{\kappa t}{a^2}\right)\frac{J_o(r\beta_n/a)}{\beta_n J_1(\beta_n)} \qquad \text{.....[2]}$$

where T_r is the relative temperature, a is the radius of a cylinder (m), and ± β_n, n=1, 2, ..., are the roots of $J_0(\beta)$=0.

Figure 1 shows the relationship between κt/a^2 and T_r for r=0, that is changes in temperature at the center of a cylinder.

Materials and Methods

1. Volcanic ash soil
   (1) soil with various water contents was packed in an Al cylinder with the bulk density of 0.66 Mg m^{-3}.
   (2) the cylinder was immersed in a constant temperature water bath (Fig. 2).
   (3) changes in temperature at the center of the cylinder were measured with 1 min. interval.

2. Simulated sand
   (1) changes in temperature at the center of a soil core were simulated using the finite element method (FEM).

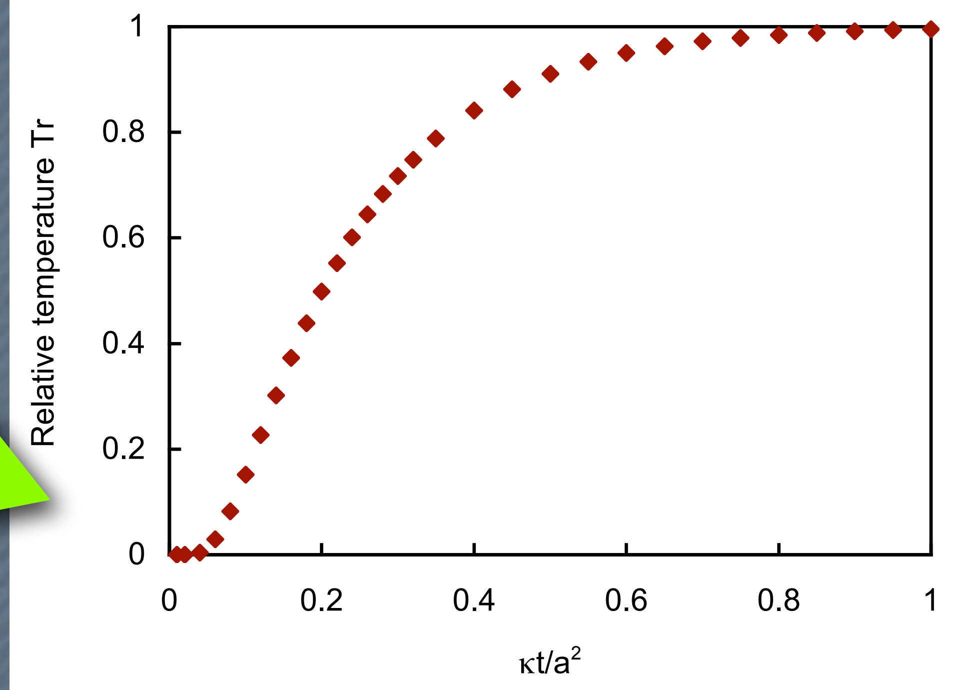

Figure 1. Relationship between κt/a^2 and T_r for r=0 in Eq. [2].

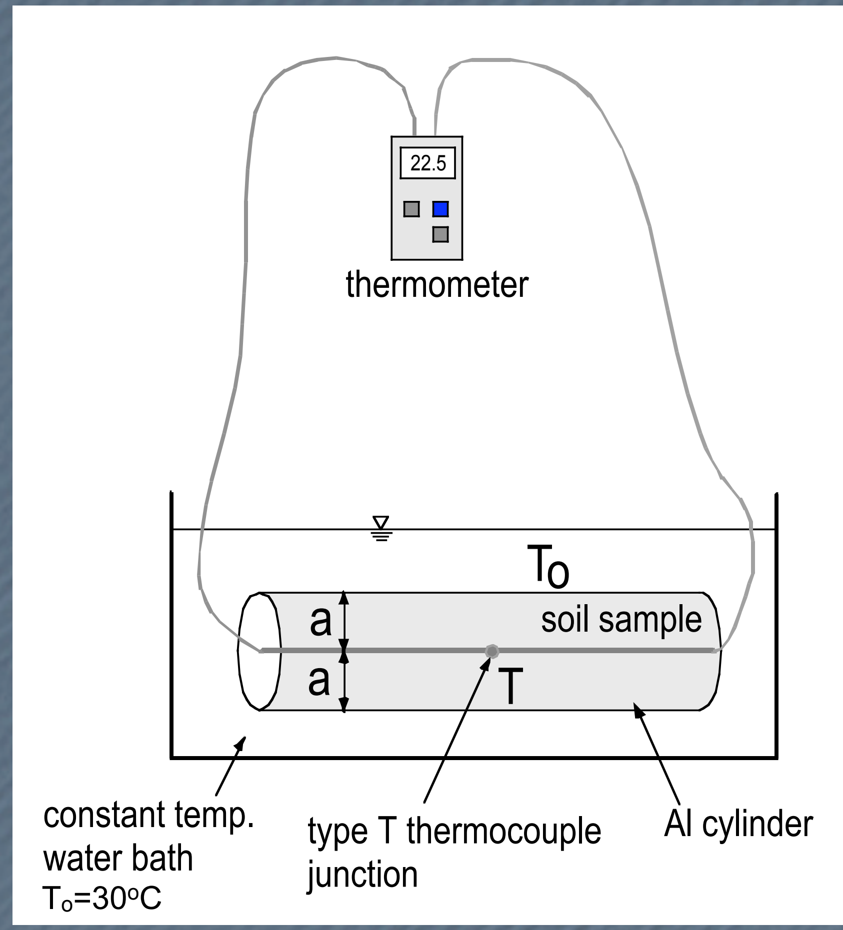

Figure 2. Schematic of experiment.

Results

1. The slope of a linear regression line for t/a^2 against κt/a^2 gave thermal diffusivity, κ (Fig. 3).

2. Thermal diffusivry for volcanic ash soil measured with the cylinder method and the DPHP method agreed well in the range between air dry and near saturation (Fig. 4).

3. Thermal diffusivity measured with the cylinder method for simulated sand agreed very well with that provided to the FEM (Fig. 5).

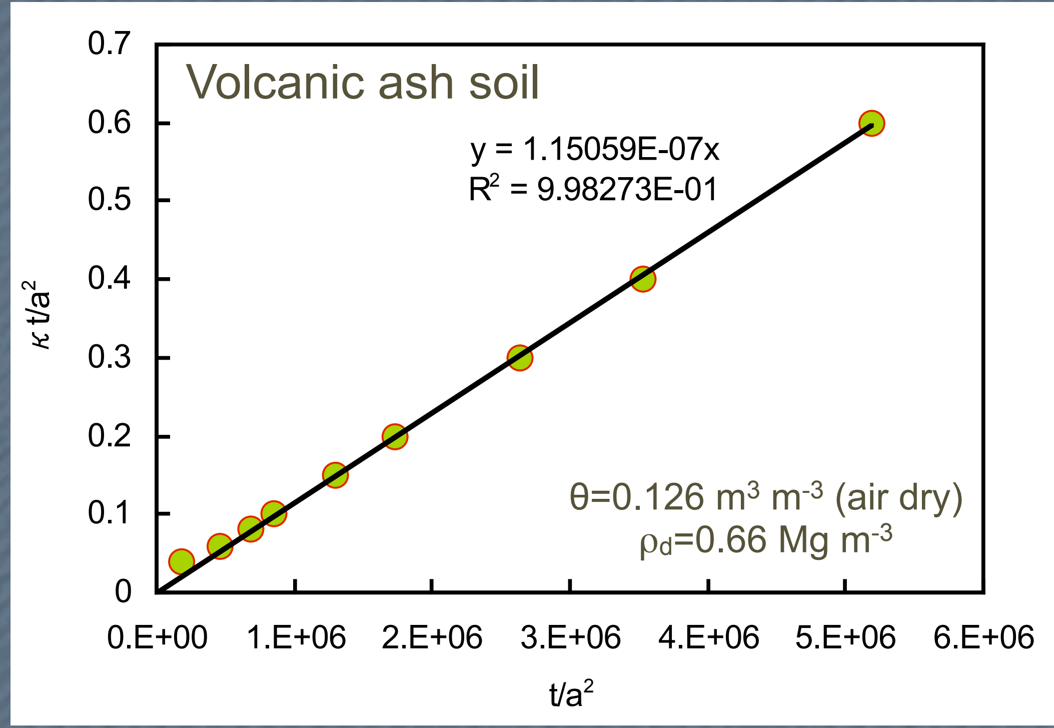

Figure 3. The slope of a linear regression line provides with thermal diffusivity, κ.

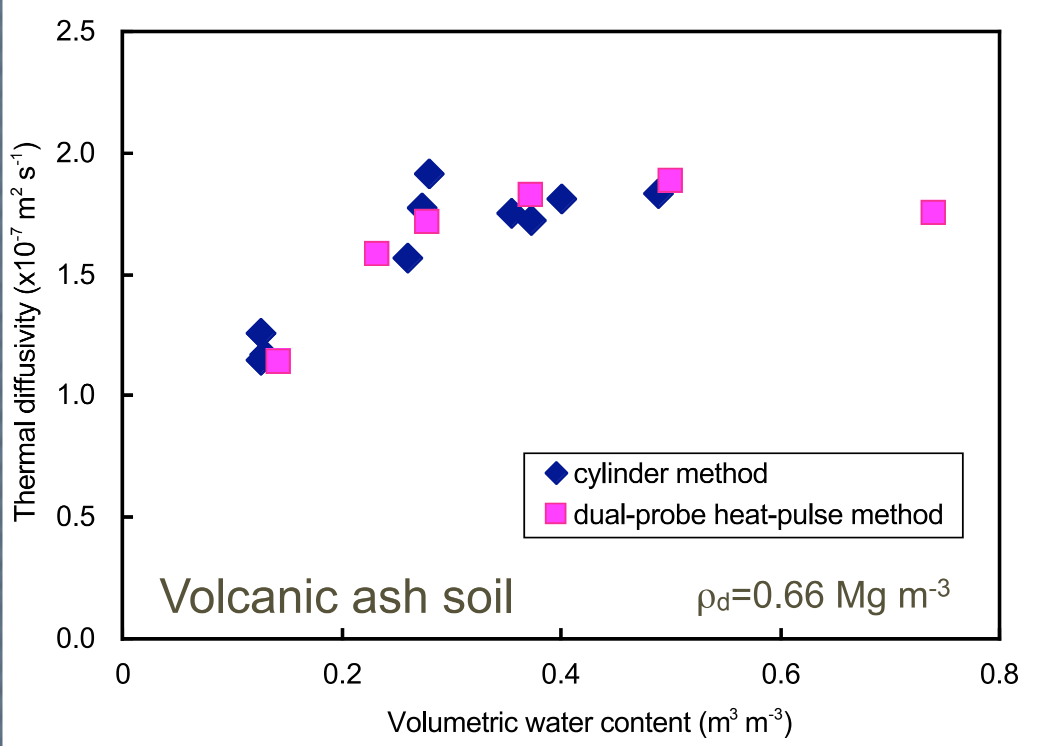

Figure 4. Thermal diffusivity, κ, for volcanic ash soil measured with cylinder and DPHP.

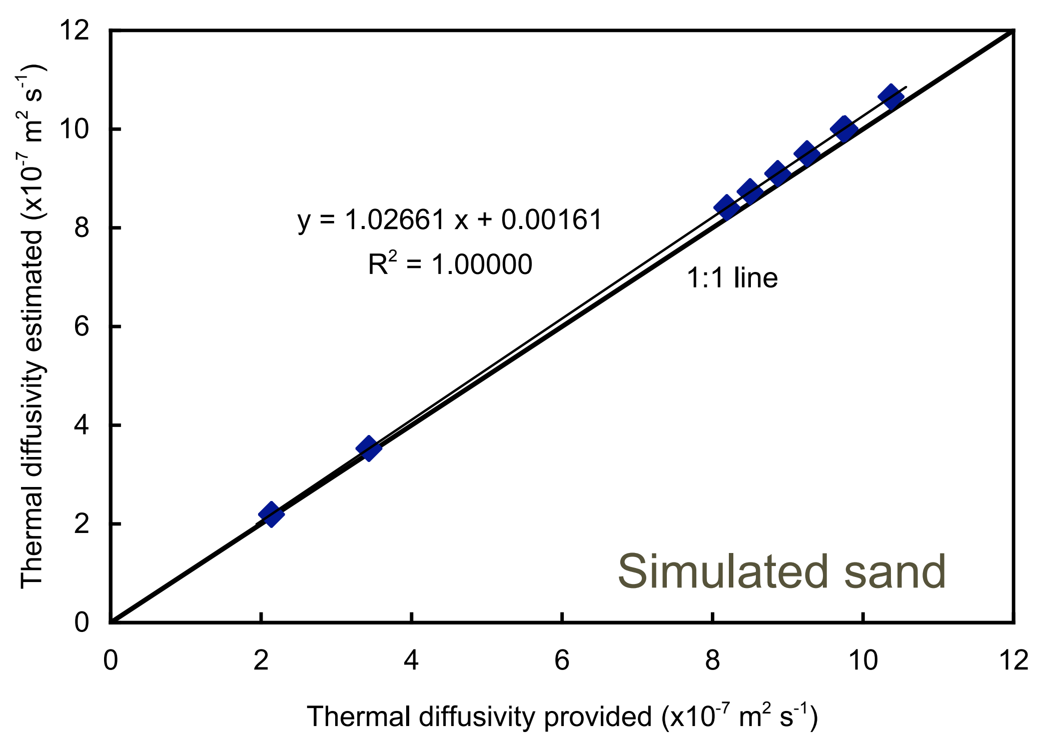

Figure 5. Thermal diffusivity, κ, for simulated sand.

References

Carslaw, H.S. and J.C. Jaeger. 1959. Conduction of heat in solids. 2nd ed. Oxford University Press, New York.

Acknowledgments

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