# 183-6: *Parallel and Perpendicular Unsaturated Hydraulic Conductivities for Layered Soils*

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## Introduction and Objective

- For saturated flow, equivalent macroscopic hydraulic conductivity is equal to arithmetic average of all individual saturated hydraulic conductivities for horizontally heterogeneous (parallel column) soils and harmonic average of all individual saturated hydraulic conductivity for vertically heterogeneous (layered) soils.
- The same average schemes were often assumed and extended to the more complex unsaturated flows. Due to the strongly non-linear relationship between the unsaturated hydraulic conductivity and the saturation, the validity of the same average schemes for unsaturated flow conditions needs to be examined.
- We investigate the appropriateness of using arithmetic and harmonic averages of hydraulic conductivity for unsaturated flows horizontal and perpendicular to the heterogeneous soil columns or layers under the steady state flow conditions involving a groundwater table.
- Relative error of calculated flux against the actual average flux of the heterogeneous soils, using various averaged hydraulic conductivity functions is also assessed. While arithmetic mean of hydraulic conductivity function performs reasonable well in simulating average flux for horizontally heterogeneous columns, harmonic mean of hydraulic conductivity function introduces quite large error in predicting the flux for the vertically heterogeneous soil layers, especially for the highly heterogeneous soils.

## Methods

Steady state flows in a layered soil formation

$$z_{i+1} - z_i = \int_{\psi_i}^{\psi_{i+1}} \frac{K_{i+1}(\psi)}{K_{i+1}(\psi) + q} d\psi$$

For parallel heterogeneous soils and Gardner exponential hydraulic conductivity function

$$q = K_S \frac{1 - e^{\alpha^*(1-h)}}{e^{\alpha^*} - 1}$$

Joint lognormal distributions for two lognormally distributed variables $K_S$ and $\alpha^*$ is

$$f(\alpha^*, K_S) = \frac{1}{2\pi\sigma_{\ln\alpha^*}\sigma_{\ln K_S}\alpha^* K_S\sqrt{1-r^2}} \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(\ln\alpha^* - \langle\ln\alpha^*\rangle)^2}{\sigma^2_{\ln\alpha^*}} - 2r\frac{(\ln\alpha^* - \langle\ln\alpha^*\rangle)(\ln K_S - \langle\ln K_S\rangle)}{\sigma_{\ln\alpha^*}\sigma_{\ln K_S}} + \frac{(\ln K_S - \langle\ln K_S\rangle)^2}{\sigma^2_{\ln K_S}}\right]\right\}$$

Average flux across the horizontally heterogeneous field (consisting of parallel columns)

$$\langle q\rangle = \int_0^\infty\int_0^\infty \frac{K_S\left[1 - e^{\alpha^*(1-h)}\right] f(K_S, \alpha^*) dK_S d\alpha^*}{e^{\alpha^*} - 1}$$

For vertically heterogeneous soils and Gardner exponential hydraulic conductivity function

$$\psi^*_{i+1} = -\frac{1}{\alpha^*_{i+1}} \ln\left\{e^{-\alpha^*_{i+1}(z^*_{i+1} - z^*_i + \psi^*_i)} - \frac{q}{K_{si+1}}\left[1 - e^{-\alpha^*_{i+1}(z^*_{i+1} - z^*_i)}\right]\right\}$$

Arithmetically averaged unsaturated hydraulic conductivity

$$\langle K(\psi)\rangle = \int_0^\infty\int_0^\infty K(\psi) f(K_S, \alpha^*) dK_S d\alpha^*$$

Harmonically averaged unsaturated hydraulic conductivity

$$\langle\langle K(\psi)\rangle\rangle = \left[\int_0^\infty\int_0^\infty \frac{f(K_S, \alpha^*)}{K(\psi)} dK_S d\alpha^*\right]^{-1}$$

Gardner exponential hydraulic conductivity function using certain types of averaged hydraulic parameters

$$K\langle +1\rangle = \langle K_S\rangle e^{-\langle\alpha^*\rangle\psi^*}$$

$$K\langle -1\rangle = \langle\langle K_S\rangle\rangle e^{-\langle\alpha^*\rangle\psi^*}$$

General *p*-norm average of hydraulic conductivity

$$K_p(\psi) = \left[\int_0^\infty\int_0^\infty K^p(\psi) f(K_S, \alpha^*) dK_S d\alpha^*\right]^{1/p}$$

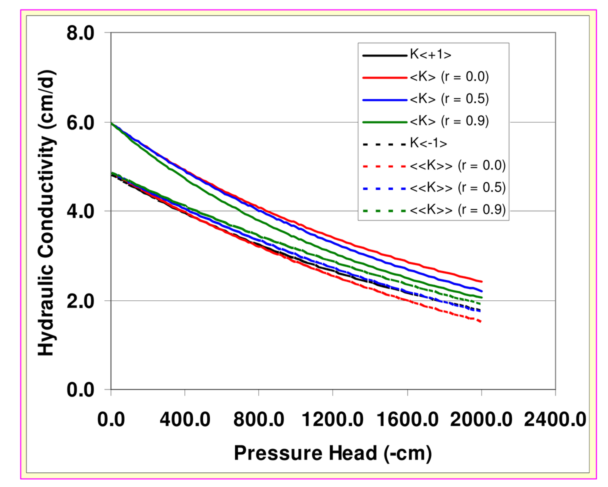
Using various averaged unsaturated hydraulic conductivity functions as hydraulic conductivity for conceptualized equivalent homogeneous soil, flux based on those averaged hydraulic conductivity functions needs to be calculated iteratively from

$$L = \int_0^{hL} \frac{K_p(\psi)}{K_p(\psi) + q_p} d\psi$$

Relative difference (error) between actual moisture flux (either <*q*> for the horizontally heterogeneous soils or *q* for vertically heterogeneous soils) and the one based on p-norm averaged unsaturated hydraulic conductivities

$$e = \frac{q_p - q_{actual}}{q_{actual}}$$

## Results



Averaged hydraulic conductivity functions for Soil A at three selected levels of correlation between $\alpha^*$ and $K_S$ (r = 0.0, 0.5, and 0.9) when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. < > indicates arithmetic mean, << >> denotes harmonic mean

**Soil A:**

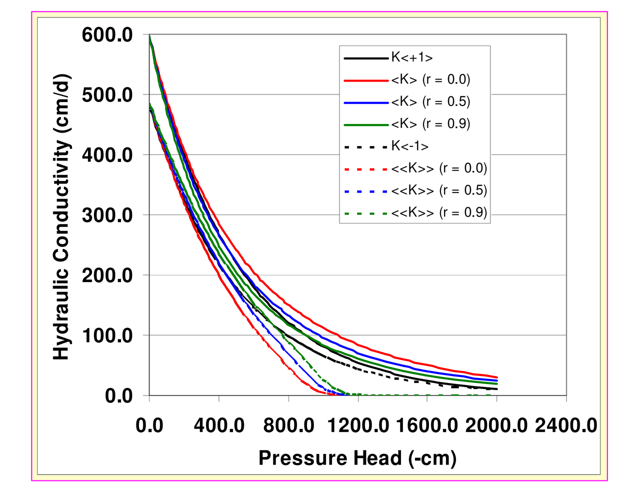
$K_S$ = 6 (cm/d),

$\alpha$ = 0.0005 (1/cm)

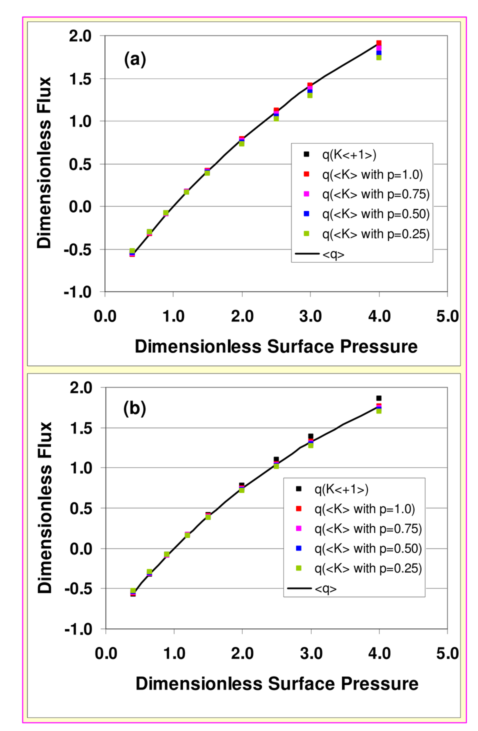
**Soil B:**

$K_S$ = 600 (cm/d),

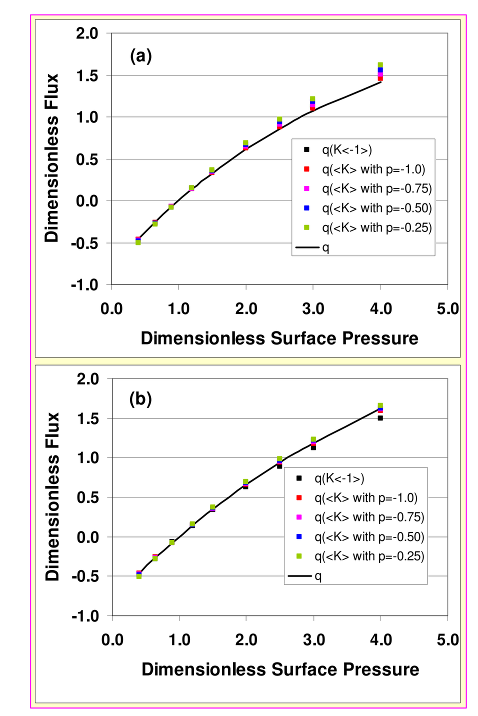
$\alpha$ = 0.002 (1/cm)



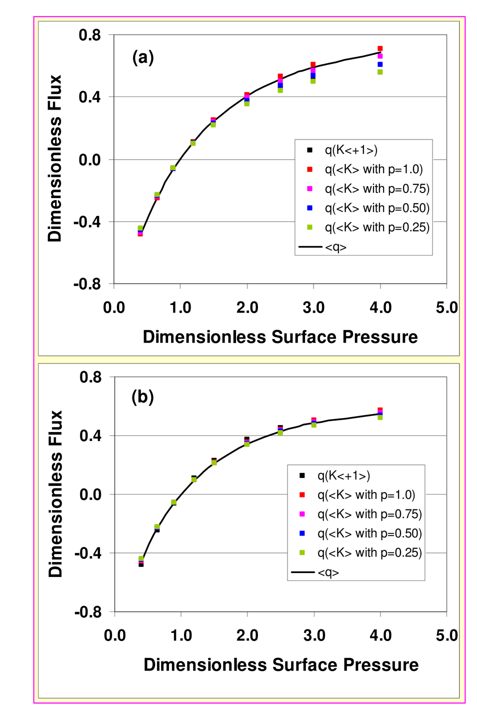
Averaged hydraulic conductivity functions for Soil B at three selected levels of correlation between $\alpha^*$ and $K_S$ (r = 0.0, 0.5, and 0.9) when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. < > indicates arithmetic mean, << >> denotes harmonic mean



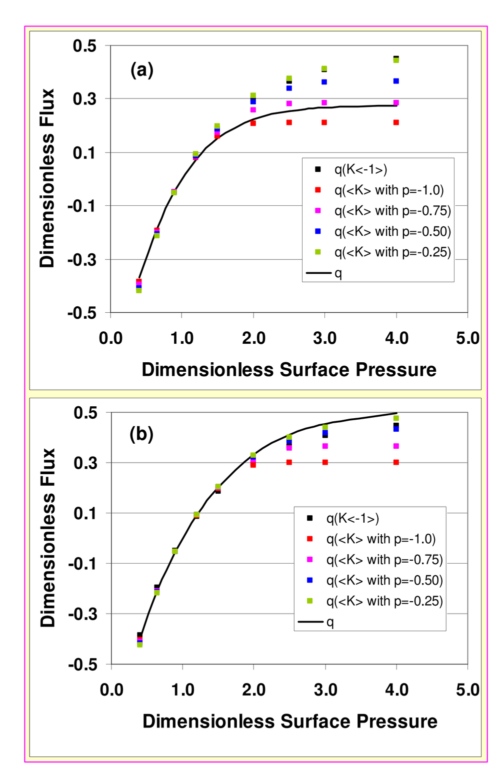
Dimensionless flux of horizontally heterogeneous case for Soil A when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. (a) *r* = 0.0, (b) *r* = 0.9.



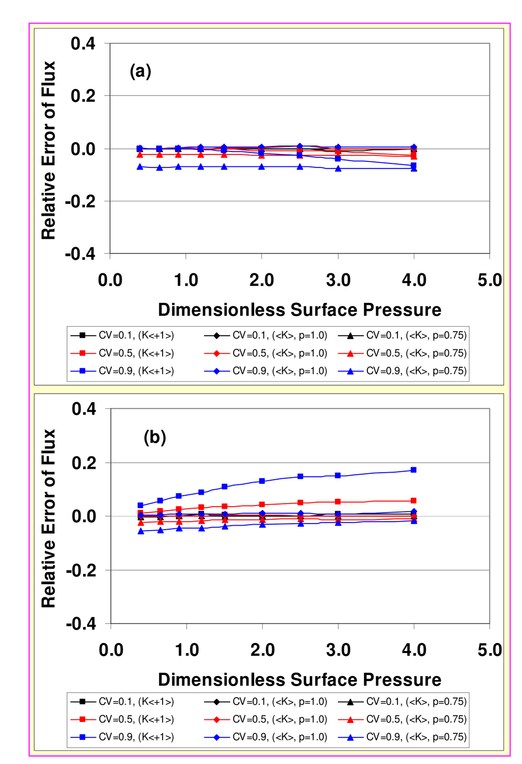
Dimensionless flux of vertically heterogeneous case for Soil A when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. (a) *r* = 0.0, (b) *r* = 0.9.



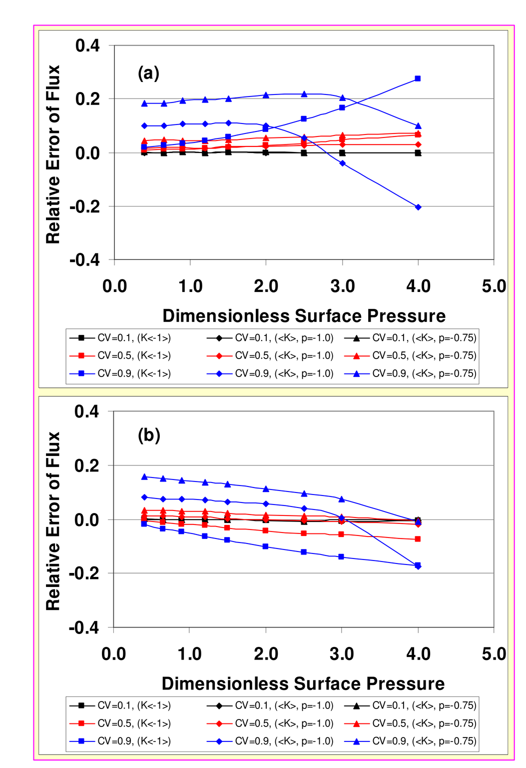
Dimensionless flux of horizontally heterogeneous case for Soil B when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. (a) *r* = 0.0, (b) *r* = 0.9.



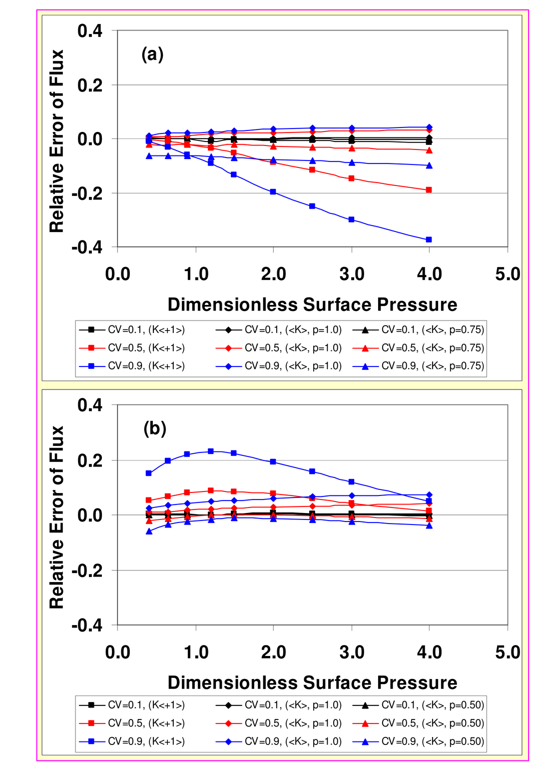
Dimensionless flux of vertically heterogeneous case for Soil B when both $CV_{\alpha^*}$ and $CV_{K_S}$ are 0.5. (a) *r* = 0.0, (b) *r* = 0.9.



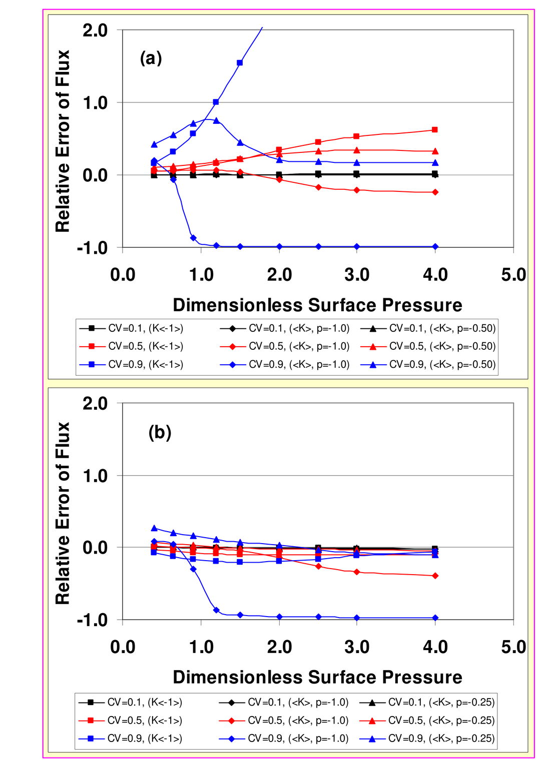
Relative error of flux (*e*) of horizontally heterogeneous case for Soil A at selected variance levels ($CV_{\alpha^*}$ and $CV_{K_S}$ values of 0.1, 0.5, and 0.9). (a) *r* = 0.0, (b) *r* = 0.9.



Relative error of flux (*e*) of vertically heterogeneous case for Soil A at selected variance levels ($CV_{\alpha^*}$ and $CV_{K_S}$ values of 0.1, 0.5, and 0.9). (a) *r* = 0.0, (b) *r* = 0.9.



Relative error of flux (*e*) of horizontally heterogeneous case for Soil B at selected variance levels ($CV_{\alpha^*}$ and $CV_{K_S}$ values of 0.1, 0.5, and 0.9). (a) r = 0.0, (b) r = 0.9.



Relative error of flux (*e*) of vertically heterogeneous case for Soil B at selected variance levels ($CV_{\alpha^*}$ and $CV_{K_S}$ values of 0.1, 0.5, and 0.9). (a) *r* = 0.0, (b) *r* = 0.9.

## Conclusions

- For unsaturated steady state flows, macroscopic unsaturated hydraulic conductivity that is between the arithmetic mean and the geometric mean of the horizontally heterogeneous soils is an appropriate in simulating average flux across heterogeneous landscape. The most appropriate p-norm value is determined by texture and heterogeneity of soils. For coarse textured soils and more heterogeneous landscape, macroscopic hydraulic conductivity should move more toward geometric mean. For vertically heterogeneous (layered) soil formations, macroscopic hydraulic conductivity would be between geometric mean and harmonic mean of heterogeneous unsaturated hydraulic conductivity.
- Arithmetic mean of unsaturated hydraulic conductivity performs better than harmonic mean of hydraulic conductivity when being used as macroscopic hydraulic conductivity in predicting average flux across horizontally heterogeneous landscape and flux in vertically heterogeneous (layered) soils. For vertically heterogeneous soils with large variances, harmonic mean of hydraulic conductivity would severely under-predict the vertical flux.
- Optimal macroscopic hydraulic conductivities for both horizontally and vertically heterogeneous soils also depend on the pressure head conditions at land surface. It is more a challenge when pressure head at the land surface is higher (i.e., dry conditions under evaporation scenario).
- Parametric correlations among hydraulic parameters are also important in determining appropriate macroscopic hydraulic conductivity for heterogeneous soils.

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