Estimating Seepage and Exit Gradients to Incised Ditches in a Homogeneous Isotropic Soil

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ABSTRACT

This poster concerns the analysis of seepage to ditches with a water level lower than the adjacent field water table and the hydraulic pressures near the ditches in order to assess their contributing role in gully development. The approach consists of analysis of the subsurface flow regime under steady state conditions using the theory of conformal transformations. First, seepage to a fully filled circular drain in a homogeneous isotropic layer overlaying an impermeable layer was analyzed (drain model). Subsequently, the case of drainage to a ditch partially filled with water and the effect of a buffer strip adjacent to the ditch on seepage reduction were determined using the same methodology (ditch model). Two approaches were considered to approximate the water level in the buffer strip: (1) a confined boundary represented by a straight line between the ditch water level and the field surface water level; and (2) a free boundary in which the groundwater level was approximated by the pressure potential relationship for flow from a ponded surface area to a sink placed at the surface at a distance equal to the width of the buffer strip. The latter analysis yielded explicit expressions of seepage, and the groundwater hydraulic and stream potential functions in terms of the spatial coordinates.

INTRODUCTION

On bottomland areas adjacent to streams, ponded water conditions and/or saturated soil profiles exist for prolonged periods during wet seasons. In those situations, appreciable amounts of water may seep to the streams where water levels are lower than the water level of the ponded surface or that of ground water. These conditions may lead to reduced bank stability and sloughing because of positive or reduced negative soil water pressures. Under these circumstances, there is the potential of incipient gully development at certain stream bank locations, chemical leaching, entrainment in flow of detached soil particles, and bank failures. From both the hydrology and water quality standpoints, it is important to know where and what the sources of water, sediment, and pollutants are and what their relative contributions to the stream system are in a watershed.

In this poster, an improved relationship will be derived using a very similar physical realization than that of Hooghoudt but based on potential flow theory to ditches partially filled with water. The models will allow both estimates of seepage for different realizations of the drainage area and estimates of the location of hydraulic aquipotentials in the proximity of the ditches that are important factors in gully development.

PHYSICAL REALIZATION

The physical realization considered is a flat land area consisting of a homogeneous, isotropic soil layer of finite thickness that overlays an impermeable layer. The soil layer is incised by ditches with circularly shaped bottoms. The semi-circular bottom part of the ditch is filled with water that is maintained at a constant water level which is lower than the groundwater table or surface water level in the surrounding field. The area between ditches is ponded as a result of prolonged rainfall or run-on from adjacent sloping land. The model further assumes a small, slightly elevated strip of land immediately adjacent to the ditch, a buffer strip, that is free of surface water. No water enters into the soil in this strip. Water seeps into the ditch by subsurface flow.

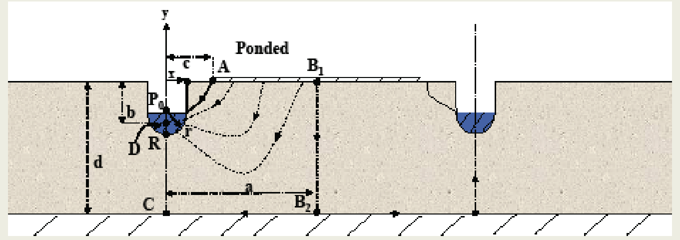

Fig. 1. A schematic representation of the flow region.

SOLUTION APPROACH: DRAIN MODEL

The solution approach often chosen for steady flow is that by conformal transformations. A brief summary of the relationships used in conformal theory are given in the Literature (Muskat and Wyckoff, 1927; van Deemter, 1950; Nehari, 1952; von Koppenfels and Stallmann, 1959; Verruijt, 1970). The flow field is described in terms of complex planes for the spatial area and the flow field in that space. The simplest version of the conformal mapping technique, as it relates to the case at hand, is to map the complex planes z and ω into the upper half of a common complex plane, say ε. Fig. 2 represents a schematic diagram in which a series of successive conformal transformations of the z-plane and ω-plane are plotted on the upper part of the ε-plane with the vertices of the z and ω-plane sharing common points on the real axis of the ε-plane. Fig. 3 presents the details of each stepwise conformal transformation and the formulae used.

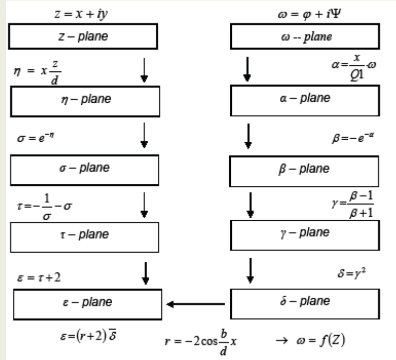

Fig. 2. A schematic representation of sequential conformal transformations of the complex z-plane and ω-plane onto the upper half of the complex ε-plane for the flow from a horizontal water table to a fully filled drain.

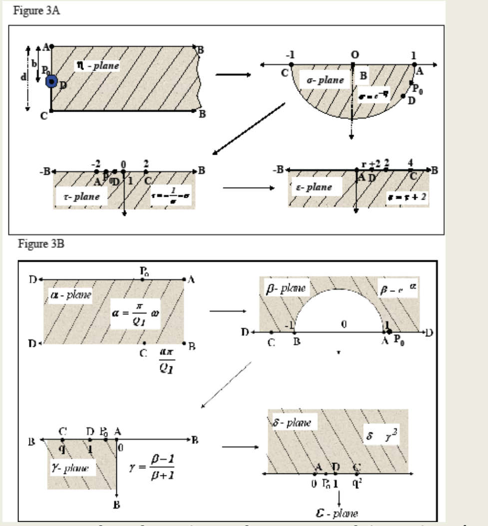

Fig. 3. Details of the sequence of conformal transformations of the z-plane (Fig. 3A) and ω-plane (Fig. 3B) onto the ε-plane.

SOLUTION (DRAIN MODEL)

Mutual substitution of the transformations yield the following close-form analytical solution for the flow field of this case, from now on called the drain model:

$$\omega = -\frac{Q_1}{\pi}\ ln\left[\frac{\sqrt{(1+r/2)}+\sqrt{1-\cosh z\frac{\pi}{d}}}{-\sqrt{(1+r/2)}+\sqrt{1-\cosh z\frac{\pi}{d}}}\right]$$

where Q1 is the total seepage or discharge into the drain and r = -2 cos bπ/d, representing a parameter that describes the relationship between the location of the drain relative to the depth of the impermeable layer.

SOLUTION (DITCH MODEL)

A similar approach was taken for the ditch model. In this case a strip of width c is introduced (see Fig. 1), in which no water is allowed to infiltrate into the soil profile. The analysis yields:

$$\omega = -\frac{Q_1}{\pi}\ ln\left[\frac{-\sqrt{cosh\frac{c}{d}\pi - cosh\frac{z}{d}\pi} - \sqrt{cosh\frac{c}{d}\pi - cos\frac{b}{d}\pi}}{-\sqrt{cosh\frac{c}{d}\pi - - cosh\frac{z}{d}\pi} + \sqrt{cosh\frac{c}{d}\pi - cos\frac{b}{d}\pi}}\right]$$

HYDRAULIC POTENTIAL FUNCTION

From the close-form analytical solution one can now calculate for each point z(x,y) the corresponding values of ω(ϕ,ψ). The corresponding expressions are:

$$\frac{\cosh\frac{c}{d}\pi - \cos\frac{y}{d}\pi\cdot\cosh\frac{x}{d}\pi}{\cosh\frac{c}{d}\pi - \cos\frac{b}{d}\pi} = \frac{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\sin^2\frac{\pi\psi}{Q_1} - 2e^{\frac{-2\pi\varphi}{Q_1}} + 1}{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\cos^2\frac{\pi\psi}{Q_1} + 2e^{\frac{-2\pi\varphi}{Q_1}} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 1}$$

and

$$\frac{\sin\frac{y}{d}\pi\cdot\sinh\frac{x}{d}\pi}{\cosh\frac{c}{d}\pi - \cos\frac{b}{d}\pi} = \frac{4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\sin\frac{\pi\psi}{Q_1} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\sin\frac{\pi\psi}{Q_1}}{e^{\frac{-4\pi\varphi}{Q_1}} - 4e^{\frac{-3\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 4e^{\frac{-2\pi\varphi}{Q_1}}\cdot\cos^2\frac{\pi\psi}{Q_1} + 2e^{\frac{-2\pi\varphi}{Q_1}} - 4e^{\frac{-\pi\varphi}{Q_1}}\cdot\cos\frac{\pi\psi}{Q_1} + 1}$$

CALCULATIONS

In this poster, all computations were performed with one data set in which the thickness d of the aquifer is 9.5 m, the depth b of the ditch relative to the ground water table is 2m, the radius r_o of the circularly shaped ditch bottom is 0.5 m, and the hydraulic conductivity per unit linear drain length is 3 m²/day. At all times the water level difference between the soil surface and the ditch was maintained at 1.5m.

RESULTS

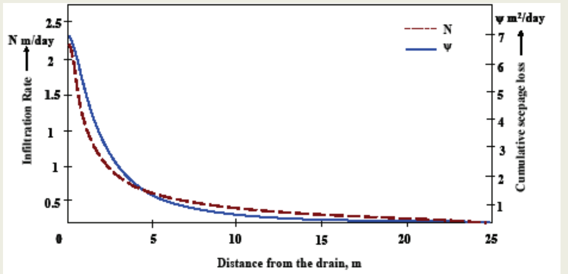

Fig. 4. Cumulative seepage loss to the drain and infiltration requirements near the drain to maintain a ponded surface.

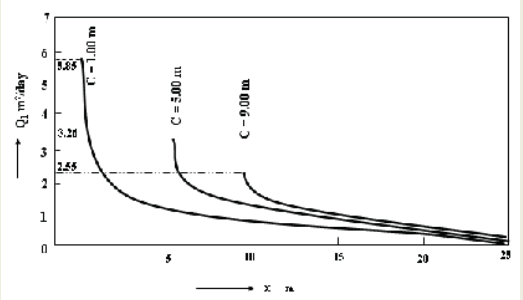

Fig. 5. Accumulated seepage loss with distance from the ditch for 3 buffer zone widths.

SUMMARY

In this poster, a relatively simple subsurface flow case was considered for which an explicit relationship was derived to predict seepage from upland areas into an incised ditch. This relationship, for the type condition considered, can readily be incorporated in watershed hydrology and water quality models to account for subsurface flow. The explicit nature of this relationship allows the expression of the hydraulic potential and streamline functions in terms of the spatial coordinates x and y which makes it possible to determine seepage and probable locations of incipient rilling and gullying, and the contribution of upland pollutant sources by subsurface flow to surface waters.

LITERATURE CITED

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