

# Estimation of Boundary Layer Conductance, Surface Water Vapor Conductance, and Net Radiation in the Penman-Monteith Equation

We dedicate this work to John Lennox Monteith who passed away on July 20, 2012, in Edinburgh, Scotland

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## Introduction

The Penman-Monteith (P-M) equation is widely applied to estimate evapotranspiration (ET) of crop fields. Currently, typical application of the P-M equation does not include stability effects in the calculation of the boundary layer heat/water vapor conductance term and assumes the surface water vapor conductance term is constant. Additionally, many weather-stations are not equipped with a net radiometer, thus net radiation is estimated from solar radiation measurements. The purposes of this study were to develop simple equations to estimate the conductance terms and test the new equations, and to show how improved estimates of net radiation can improve ET estimation.

The two-source surface energy balance model described in Blonquist et al. (2009), a modified form of the model of Norman et al. (1995), was used to develop equations for calculating boundary layer heat/water vapor conductance and reference surface water vapor conductance. A well-watered alfalfa canopy with height of 0.5 m and leaf area index of 4.5 m<sup>2</sup> m<sup>-2</sup> was assumed in model calculations. A data set collected over well-watered alfalfa, where ET was measured with a weighing lysimeter, was used to verify the newly developed equations.

## Penman-Monteith Equation

The P-M equation is derived from the surface energy balance and provides ET estimates from net radiation, vapor pressure deficit, and the conductance terms:

$$\lambda E = \frac{\Delta(R_n - G) + g_s C_p (e_s - e_a)}{\Delta + \gamma \left( 1 + \frac{g_b}{g_s} \right)}$$

where  $\lambda E$  is latent heat flux [W m<sup>-2</sup>] (ET is calculated from  $\lambda E$  by dividing by the latent heat of vaporization  $\lambda$  [J mol<sup>-1</sup>] and multiplying by the molar mass of water, 0.018 kg mol<sup>-1</sup>),  $R_n$  is net radiation [W m<sup>-2</sup>],  $G$  is soil heat flux [W m<sup>-2</sup>],  $C_p$  is heat capacity of air [J mol<sup>-1</sup> C<sup>-1</sup>],  $e_s$  is saturation vapor pressure of air [kPa],  $e_a$  is actual vapor pressure of air [kPa],  $\Delta$  is the slope of the saturation vapor pressure-temperature relationship [kPa C<sup>-1</sup>],  $\gamma$  is the psychrometric constant [kPa C<sup>-1</sup>],  $g_b$  is canopy boundary layer conductance to heat and water vapor (often called aerodynamic conductance) [mol m<sup>-2</sup> s<sup>-1</sup>], and  $g_s$  is surface water vapor conductance [mol m<sup>-2</sup> s<sup>-1</sup>]. ET is largely driven by  $R_n$  and vapor pressure deficit ( $e_s - e_a$ ), as shown by the P-M equation. Thus, ET estimates from the P-M equation are sensitive to  $R_n$ . The sensitivity to  $g_b$  and  $g_s$  increases as  $e_s - e_a$  increases and as wind speed increases.

## Boundary Layer Heat/Water Vapor Conductance Equation

Boundary layer heat/water vapor conductance ( $g_b$ ) is dependent on wind speed, surface roughness, and stability in the layer of air near the surface. Calculation of  $g_b$  is based on the shape of the wind speed, temperature, and humidity profiles in the surface boundary layer:

$$g_b = \frac{u \rho_{mol} k^2}{\ln \left( \frac{z_m - d}{z_m} \right) - y'_m \left[ \ln \left( \frac{z_{Ta} - d}{z_{Ta}} \right) - y'_{Tv} \right]}$$

where  $u$  is wind speed [m s<sup>-1</sup>],  $\rho_{mol}$  is molar density of air [mol m<sup>-3</sup>],  $k$  is the von Karman constant,  $z_m$  and  $z_{Ta}$  are wind speed and air temperature measurement heights [m], respectively,  $d$  is zero plane displacement height [m],  $z_m$  and  $z_{Ta}$  are roughness lengths for momentum and heat/water vapor [m], respectively, and  $y'_m$  and  $y'_{Tv}$  are stability parameters for momentum and heat/water vapor, respectively. The equation for neutral conditions, used in typical applications of the P-M equation, does not include  $y'_m$  and  $y'_{Tv}$ . To derive an equation for  $g_b$  that includes the effects of stability,  $g_b$  from the two-source model, which includes the stability parameters, was divided by  $g_b$  from the equation for neutral conditions to produce the data shown in Figure 1 for stable and unstable conditions. The neutral equation was then modified by deriving empirical coefficients that produced the best fit to the data in Figure 1. ET predictions from the P-M equation, assuming neutral conditions and using the modified equations, indicate potential improvement when stability is accounted for (Figure 2); stability, stable or unstable, was determined by calculating the bulk Richardson number). However, more data at low wind speeds should be tested.

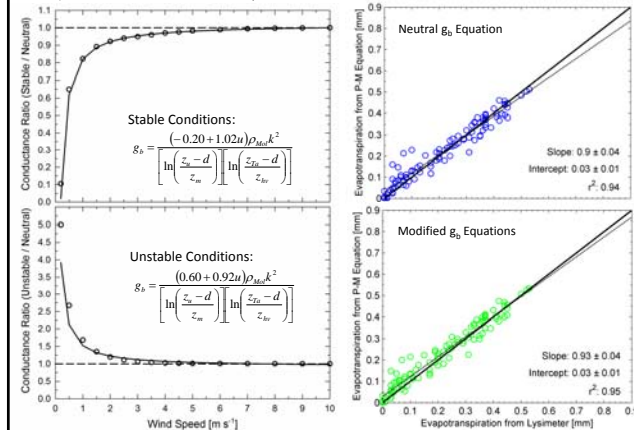


Fig. 1: Stable or unstable  $g_b$  divided neutral  $g_b$  as a function of wind speed.

Fig. 2: ET from the P-M equation compared to measured ET.

## Surface Water Vapor Conductance Equation

For vegetated surfaces, surface water vapor conductance ( $g_s$ ) is highly dependent on plant water status. Typical applications of the P-M equation assume a constant reference  $g_s$ . However,  $g_s$  for well-watered plants should vary with light level and leaf area index (LAI). To derive an equation for reference  $g_s$ , canopy stomatal conductance from the two-source model was plotted versus incoming shortwave radiation (SW<sub>i</sub>) at four different levels of cloudiness (Figure 3), characterized by the ratio of SW<sub>i</sub> to clear sky SW<sub>i</sub> (SW<sub>i</sub>/SW<sub>c</sub>). Cloudiness is an indirect indicator of direct versus diffuse radiation, which influences the slope of the relationship between canopy stomatal conductance and SW<sub>i</sub>. The slopes of the lines were then plotted versus SW<sub>i</sub>/SW<sub>c</sub> to allow derivation of an equation to predict  $g_s$ /SW<sub>i</sub>, and subsequently  $g_s$  (Figure 3). ET predictions from the P-M equation, assuming constant  $g_s$  and using the modified equations, indicate improvement when  $g_s$  varies with light and leaf area (Figure 4).

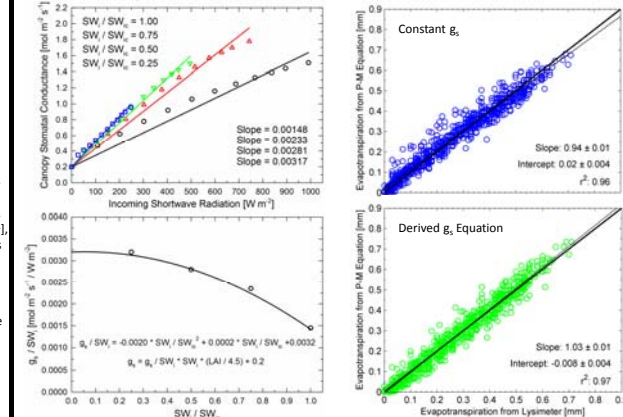


Fig. 3: Modeled  $g_s$  as a function of shortwave radiation and slope ( $g_s$  / SW<sub>i</sub>) as a function of cloudiness (SW<sub>i</sub> / SW<sub>c</sub>).

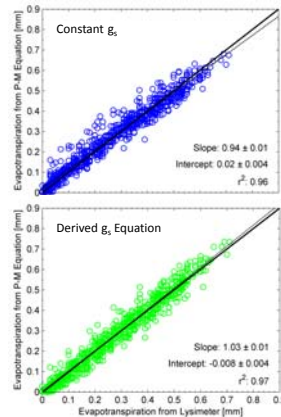


Fig. 4: ET from the P-M equation compared to measured ET.

## Net Radiation Estimation

Ideally, net radiation ( $R_n$ ) is a measured variable, but many ET weatherstations are not equipped with a net radiometer, in which case  $R_n$  is predicted from pyranometer measurements of SW<sub>i</sub>, an assumed albedo, and modeled value of net longwave radiation. To improve  $R_n$  estimates, outgoing longwave radiation (LW<sub>o</sub>) can be determined with an infrared radiometer measurement of surface temperature. Then, incoming longwave radiation (LW<sub>i</sub>) is the only variable that must be modeled in order to determine  $R_n$ . Estimates of  $R_n$ , and subsequently ET, were more accurate when LW<sub>o</sub> was measured along with SW<sub>i</sub> (Figures 5 and 6).

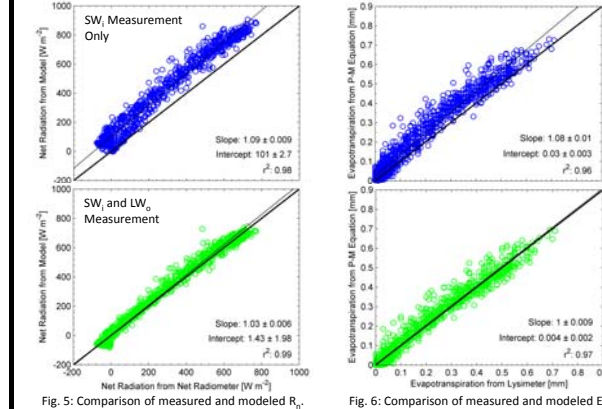


Fig. 5: Comparison of measured and modeled  $R_n$ .

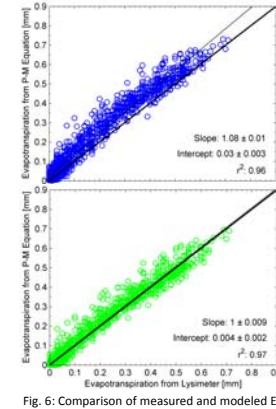


Fig. 6: Comparison of measured and modeled ET.

## Summary

- Modified conductance equations and inclusion of surface temperature measurements for  $R_n$  calculation improved ET estimates with the P-M equation when compared to ET measurements from a weighing lysimeter.
- The conductance equations are easy to apply and can be used in routine applications of the P-M equation, or in recursive calculations of ET, recently suggested as an alternative to the P-M equation.