Chen, Yong-Lin[1], Jet-Chau Wen[2,4], Tian-Chyi Yeh[3], Shao-Yang Huang[4]
[1]Graduate School of Engineering Science and Technology, National Yunlin University of Science and Technology, Douliu, Yunlin, Taiwan.
[2]Department of Environmental & Safety Engineering, National Yunlin University of Science and Technology, Douliu, Yunlin, Taiwan.
[3]Department of Hydrology and Water Resources, The University of Arizona, Arizona, USA.
[4]The Research Center for Soil & Water Resources and Natural Disaster Prevention, National Yunlin University of Science and Technology, Douliu, Yunlin, Taiwan.

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Reproducibility of The Estimation Using Hydraulic Tomography Analysis With In Situ Pumping Test Data

Abstract

Although Hydraulic Tomography (HT) has been employed to determine the heterogeneous hydraulic properties, there is no particular study yet on the reproducibility of the in-situ hydraulic properties. In this paper, two comprehensive sequential pumping tests on a single site in two different years, 2004 and 2010, are conducted and the drawdown data are analyzed using the specific drawdown (s/Q) to demonstrate the reproducibility of the drawdown and, subsequently, the hydraulic properties.
The results of the pumping tests show that the reproducibility of the drawdown at the single site is validated as the statistical error and the correlation coefficient between the specific drawdown-time data in 2004 and 2010 are < 0.005 and > 0.9, respectively.
The performance statistics of the estimated transmissivity (*T*) by Steady State Hydraulic Tomography (SSHT) indicates that the mean absolute error (L1 norm) is 0.001, the root mean square error (L2 norm) is 0.002, and the correlation coefficient (COR) is 0.424. The performance statistics of the estimated transmissivity (*T*) by Transient Hydraulic Tomography (THT) shows that the values for L1, L2 and COR are 0.003, 0.006 and 0.515, respectively while on the estimated storativity (*S*), the values for L1, L2 and COR are 0.013, 0.129 and 0.234, respectively. The result of statistics show that the estimated transmissivity (*T*) and the estimated storativity (*S*) have similar trends by both of SSHT and THT between 2004 and 2010. In addition to, the variogram analysis by the exponential regression also suggests that there is a correlation between the estimated fields in different years by both of SSHT and THT. Consequently, the reproducibility of the heterogeneous hydraulic properties is confirmed. The reproducibility of the in-situ hydraulic properties can be demonstrated by above two results of analysis.
Additionally, the performance statistics on the predicted drawdown by SSHT show that the values for L1, L2 and COR are 0.175, 0.211 and 0.417, respectively whereas L1, L2 and COR are 0.102, 0.168 and 0.728, respectively in the case of the predicted drawdown by THT in 2004. Then, the performance statistics on the predicted drawdown by SSHT show that the values for L1, L2 and COR are 0.167, 0.196 and 0.771, respectively whereas L1, L2 and COR are 0.100, 0.143 and 0.895, respectively in the case of the predicted drawdown by THT in 2010. The results of validation between the drawdowns by SSHT and THT in 2004 and 2010, it is noticed that if more pumping test data sets employed, a more accurate predicted drawdown can be generated, and the results are like Huang et al., [2011] mentioned the methods of validation with some characteristics.
Based on the result of the cross validation is significantly dependent on the average pumping rate. In the comparison between SSHT and THT on the results of estimated fields, HT also appears to be robust and accurate.
Keywords: reproducibility, sequential pumping test, hydraulic tomography

Introduction

Groundwater is a country's most important natural resource. Aquifer tests represent the most common approach to obtain the hydraulic properties in an aquifer. Recently, some studies (Li et al., 2008; Cardiff, et al., 2009; Illman et al., 2009; Wen, et al. , 2010; Huang et al., 2011) have confirmed that in-situ aquifers may appear to be heterogeneous. In general, the approximate analytical solution assumes the homogeneity of the aquifer in order to analyse the hydraulic properties, such as Theis method or Cooper-Jacob method. However, this assumption does not comply with the existing aquifer heterogeneity characteristics. Yeh et al. [1993] proposed the concept of Hydraulic Tomography (HT) and developed a numerical program on the basis of HT, named Variably Saturated Flow and Transport utilizing the Modified Method of Characteristics in 2D, VSAFT2).

However, the aforementioned literature each article only mentioned only uses a pumping well during a pumping test. Never one used in sequential pumping tests to prove the reproducibility of distributions of heterogeneous hydraulic properties. There are many uncertainties, use the same pumping wells doing pumping test in the same site. Therefore, we want to validate the drawdown has reproducibility is very difficult.

The purpose of this study is to prove reproducibility of drawdowns of monitoring wells of sequential pumping tests at the same location and to investigate the reproducibility of the distributions of hydraulic properties of the sequential pumping tests using HT in the two different years (2004 and 2010).

Material and Method

◈ Description of the Field Site

In this study, the sites were located on National Yunlin University of Science and Technology (NYUST), and located in the western central part of Taiwan. The site is located at the Cho-shui River alluvial fan. Four composition materials of the aquifer: gravel, sand, fine sand and clay in aquifers. At the depth of 20.0 m below the surface, soil properties are almost identical to clay's properties and the clay layer can be assumed as an impervious layer for the aquifer.

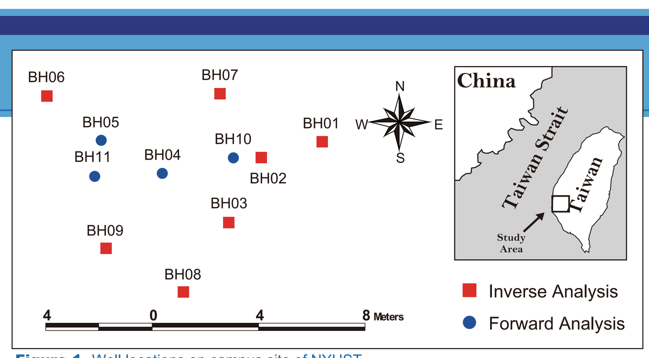

Figure 1. Well locations on campus site of NYUST.
(Note: The data from seven pumping tests (e.g. red square well) used in inverse analysis; The data from four pumping tests (e.g. Blue circle well) used in forward analysis).

◈ Sequential Pumping Test

● Introduce Sequential Pumping Test

Table 1. The setting conditions of sequential pumping tests.

Year	Date	Pumping rate range (m^3/s)	Sstatic water level of arithmetic mean (m)
2004-SPT	February, 2004-June, 2004	Between 8.00×10^{-5} and 2.06×10^{-4}	43.8
2010-SPT	August, 2010-February, 2011	Between 1.09×10^{-4} and 1.50×10^{-4}	46.5

● The Specific Drawdown Analysis (s/Q) of Pumping Test

Since the uncertainties of boundaries and domain of site in initial time are existed, so we use the specific drawdown (s/Q) analysis to view the reproducibility between 2004 and 2010. In this study, we have to reconstruct the reproducibility of unsteady drawdowns of the other ten observation well in same pumping wells from pumping tests at the same site in different times. Then, we use the specific drawdown represent the unsteady drawdowns of the same observation well of one unit pumping rate with the same pumping well. The specific drawdown following equation:

$$\bar{s}_{i,j}^{k}(t)=\frac{s_{i,j}^{k}}{Q_i^k} \quad \text{......................................(1)}$$

The $\bar{s}_{i,j}^{k}(t)$ means the specific drawdown of the *i* well as pumping well, and the one unit pumping rate for the *j* well as observation well in the *t* time from the execution of sequential pumping test in the *k* year.

The $s_{i,j}^{k}$ means the drawdown of the i well as pumping well, and the pumping rate as Qi for the j well as observation well in the *t* time from the execution of sequential pumping test in the *k* year

The Q_i^k means the pumping rate (constant flow) of the *i* well as pumping well from the execution of sequential pumping test in the *k* year.

where
i : the well numbers of pumping well from the execution of sequential pumping test. (i.e., *i* = 1, 2, 3, …, 11).
j : the well numbers of other observation wells except as pumping well from the execution of sequential pumping test. (i.e., *j* = 1, 2, 3, …, 11, but *i* ≠ *j*).
k : the execution year of sequential pumping test. (e.g. in this study, two different years, i.e., *k* = 2004 and 2010).

◈ Numerical Experiment ● 2-D Hydrogeological Numerical Modeling

In this study, the groundwater fluid simulation was executed by VSAFT2 to generate a distribution field of 2-D plane heterogeneous hydraulic properties. The numerical mode of VSAFT2 was on the basis of the numerical program developed by Yeh et al. [1993]. The data of pumping test performed based on the theory of groundwater flow movement divided into both steady-state (late time) and transient (early time) data is use in SSHT and THT. As shown Figure 2, the 2-D Hydrogeological numerical modeling was established in accordance with the actual scale of the site which was 51 m × 51 m with grid element size of 1 m^2. Therefore, in order to reduce the uncertainties of boundary in the numerical modeling, the results will only discuss the mode field of 21 m × 21 m area. As shown Table 2, the input parameters for VSAFT2 modeling.

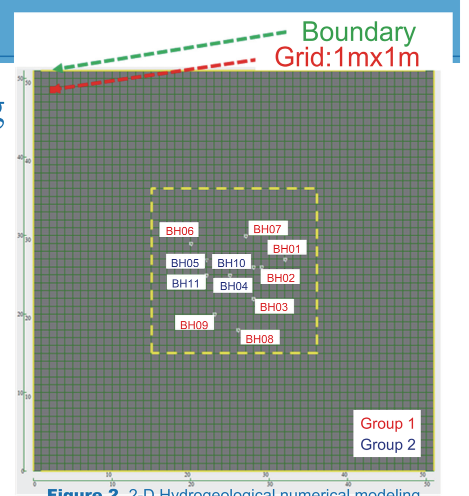

Figure 2. 2-D Hydrogeological numerical modeling.

Table 2. The input parameters for VSAFT2 modeling.

Conditions	Grids(m)	Initial and Boundary head (m)	GMT (m^2/day)	Variance of GMT (m^4/day^2)	GMS	Variance of GMS
2004		43.8	25.2	268	1.30E-03	8.71E-06
2010	51 × 51	46.5	15.3	55.8	1.34E-03	7.28E-06

(GMT: Geometric Mean of Transmissivity, GMS: Geometric Mean of Storativity, note: Storativity is non-dimensional).

● Defined Four Cases

This study in order to explores the reproducibility of heterogeneous hydraulic properties and predicted drawdown, so defines four Cases for HT (SSHT and THT) analysis the sequential pumping test in 2004 and 2010. As shown in Table 3, we defined 2004 SSHT as Case 1(a), 2010 SSHT for Case 1(b), 2004 THT for Case 2(a), and 2010 THT for Case 2(b).

Table 3. Define four case to view the reproducibility.

Methods	Year	Cases	Estimates res ults
SSHT	2004 2010	Case 1 (a)-2004 SSHT Case 1 (b)-2010 SSHT	Transmissivity
THT	2004 2010	Case 2 (a)-2004 THT Case 2 (b)-2010 THT	Transmissivity and Storativity

◈ Performance Statistics

This study using the mean absolute error (L1 norm), root mean square error (L2 norm) and standard coefficient of correlation (COR) to calculate the error and correlation between x and y as following:

$$L1=MAE=\frac{\sum\left(\left|x-y\right|\right)}{n} \quad \text{........................(2)}$$

$$L2=RMSE=\sqrt{\frac{\sum(x-y)^2}{n}} \quad \text{........................(3)}$$

where the random variables x and y denoted specific drawdowns, transmissivities, storativity and water levels, and n is the total number of random variables (x or y).

The correlation coefficient (COR)

$$\rho_{xy}=\frac{\operatorname{cov}\left[(x-\mu_x)(y-\mu_y)\right]}{\sqrt{V(x-\mu_x)^2}\sqrt{V(y-\mu_y)^2}}; \quad -1\le\rho_{xy}\le 1 \quad \text{........................(4)}$$

Results

◈ Geological Properties at Field Site

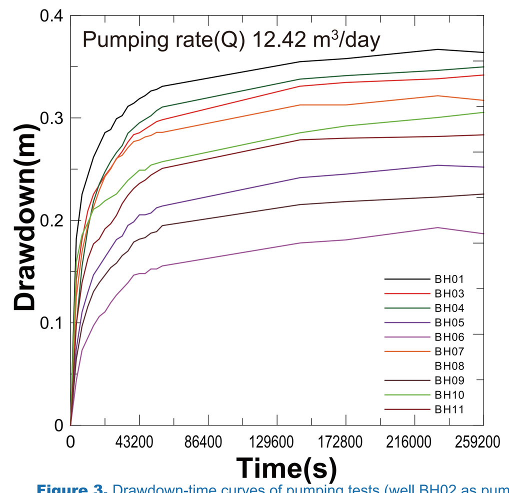

Figure 3. Drawdown-time curves of pumping tests (well BH02 as pumping well).

◈ Reproducibility of the Drawdown of Pumping tests

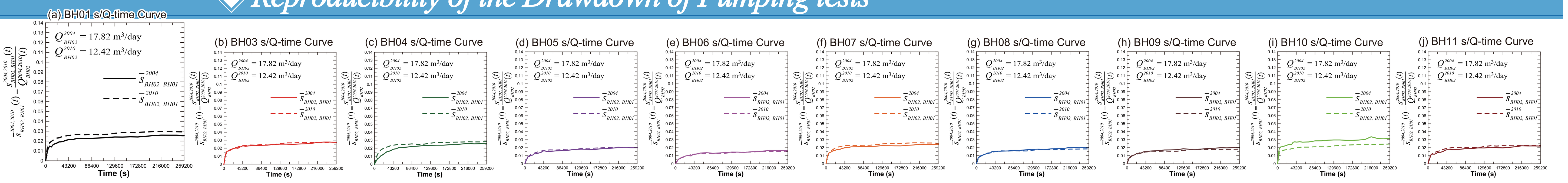

Figure 4. Comparison of s/Q curves of sequential pumping tests between 2004 and 2010 (pumping on well BH02)

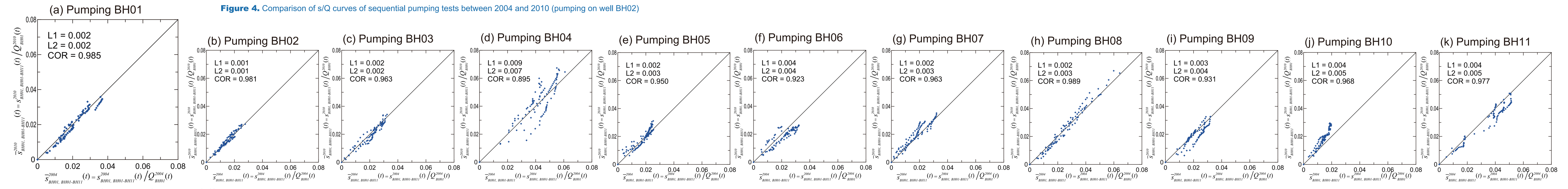

Figure 5. Comparison of correlation of s/Q between 2004 and 2010.

Table 4. Statistics of the comparison of the specific drawdown (s/Q).

Statistics / Pumping Well No.	L1 (day/m^2)	L2 (day/m^2)	COR
BH01	0.002	0.002	0.985
BH02	0.001	0.001	0.981
BH03	0.002	0.002	0.963
BH04	0.009	0.007	0.895
BH05	0.002	0.003	0.950
BH06	0.004	0.004	0.923
BH07	0.002	0.003	0.963
BH08	0.002	0.003	0.989
BH09	0.003	0.004	0.931
BH10	0.004	0.005	0.968
BH11	0.004	0.005	0.977

◈ Comparison of the Results of Estimation

● The reproducibility of estimated T fields by SSHT

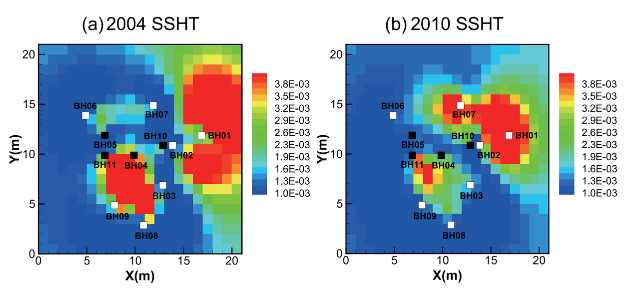

Figure 6. Estimated T fields by SSHT using data of sequential pumping test of 2004 and 2010.

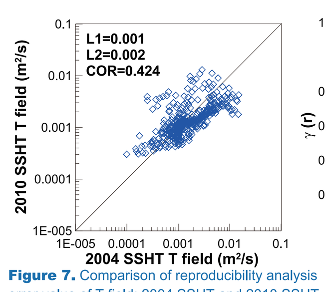

Figure 7. Comparison of reproducibility analysis error value of T field: 2004 SSHT and 2010 SSHT.

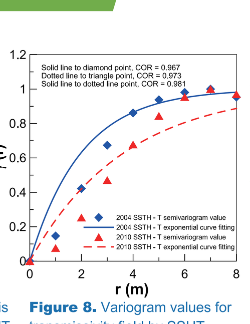

Figure 8. Variogram values for transmissivity field by SSHT.

Note: The points represent the result of variogram analysis. (2004: Blue diamond;2010: Red triangle); the curve represents the exponential fitting values. (2004: The solid blue line; 2010: Red dotted line)

● The reproducibility of estimated T and S fields by THT

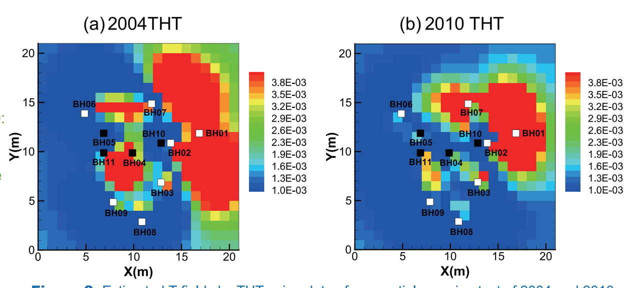

Figure 9. Estimated T fields by THT using data of sequential pumping test of 2004 and 2010.

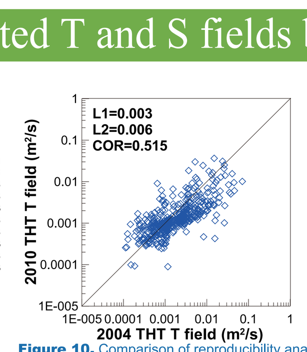

Figure 10. Comparison of reproducibility analysis error value of T field: 2004 THT and 2010 THT.

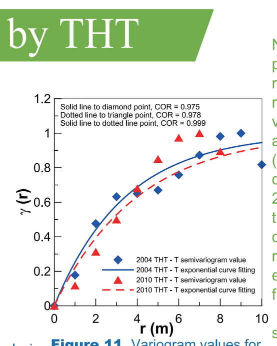

Figure 11. Variogram values for transmissivity field by THT.

Note: The points represent the result of variogram analysis. (2004: Blue diamond; 2010: Red triangle); the curve represents the exponential fitting values. (2004: The solid blue line; 2010: Red dotted line)

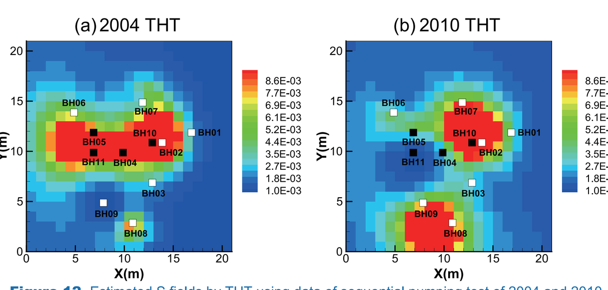

Figure 12. Estimated S fields by THT using data of sequential pumping test of 2004 and 2010.

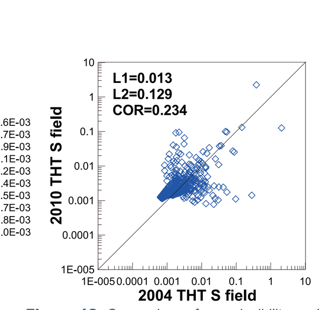

Figure 13. Comparison of reproducibility analysis error value of S field: 2004 THT and 2010 THT.

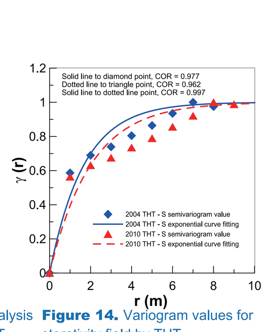

Figure 14. Variogram values for storativity field by THT.

Note: The points represent the result of variogram analysis. (2004: Blue diamond; 2010: Red triangle); the curve represents the exponential fitting values. (2004: The solid blue line; 2010: Red dotted line)

◈ Validation of Estimates

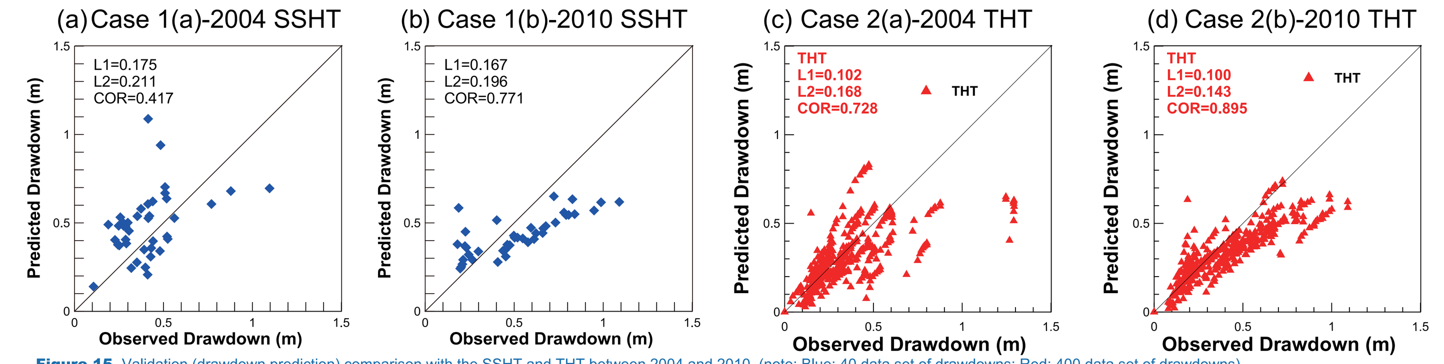

Figure 15. Validation (drawdown prediction) comparison with the SSHT and THT between 2004 and 2010. (note: Blue: 40 data set of drawdowns; Red: 400 data set of drawdowns)

◈ Cross Validation

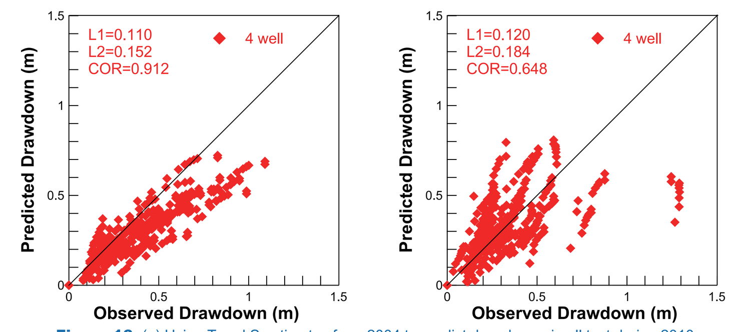

Figure 16. (a) Using T and S estimates from 2004 to predict drawdowns in all test during 2010; (b) Using T and S estimates from 2010 to predict drawdowns in all test during 2004.

Discussion and Conclusion

In this study, we obtain the following conclusions: **1.** this study demonstrates that the 2004 and 2010 sequential pumping tests show reproducible characteristics; **2.** the sequential pumping test data by HT in 2004 and 2010 can be used to represent hydraulic properties on field of testing sites on NYUST; **3.** SSHT and THT indicate that HT has singularity; **4.** We can use the result of cross validation to predict the other drawdowns of sequential pumping test next time.

We have also confirmed the feasibility of application of HT on in-situ sequential pumping tests with the assumption of heterogeneous aquifer and stated the effectiveness of VSAFT2 for estimating heterogeneous hydraulic properties. In addition to determination of the reproducibility in the sequential pumping tests by HT, one of our future research directions is to integrate the pumping test data with HT to prove the existence of the mathematical theory of reciprocity. If we take advantage of the approach reported in this study to collect hydraulic properties, it will be a significant breakthrough to the groundwater management.

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