Effective Unsaturated Hydraulic Conductivity and Characteristic Length of Layered Soils Considering Steady-State Evaporation

Morteza Sadeghi[1], Scott B. Jones[1], Markus Tuller[2], Mohammad R. Gohardoust[3]

1) Plants, Soils, and Climate Dept., Utah State University 2) Soil, Water, and Environmental Science Dept., The University of Arizona
3) Water Engineering Dept., Sari Agricultural Sciences and Natural Resources University, Sari, Iran

Introduction

The maximum water table depth, D_{max}, that sustains a hydraulic connection with the soil surface during evaporation is termed "characteristic length of evaporation" (Lehmann et al., 2008) and provides a length scale, which is correlated with soil hydraulic properties.

In this paper a new general solution for D_{max} in coarse-textured homogeneous and layered soils under steady-state evaporation from a water table is presented. Here, "general" means that the solution is not restricted to a specific form of the unsaturated hydraulic conductivity function.

The solution provides an alternative method for determination of unsaturated hydraulic conductivity of homogeneous soil profiles. Also it offers a new approach to the effective or upscaled unsaturated hydraulic conductivity of layered soil profiles.

Theoretical Considerations

Buckingham–Darcy law:

$$D_{max} = \int_0^{h_{max}} \frac{K(h)}{K(h)+e}\,dh \qquad (1)$$

D_{max}: characteristic length (Fig. 1)
h: pressure head
h_{max}: pressure head at drying front
e: steady-state evaporation rate
K: unsaturated hyd. conductivity

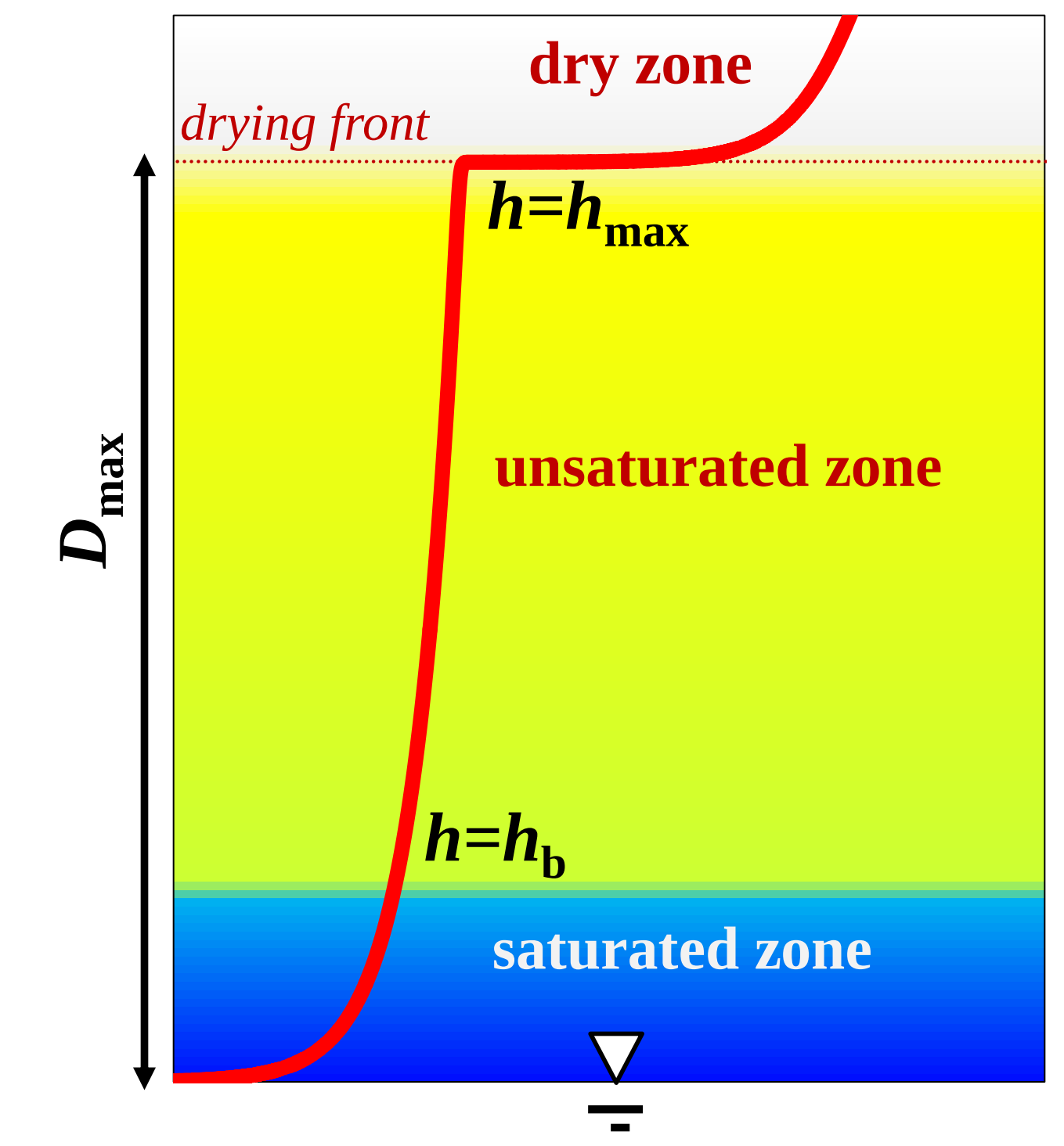


Fig. 1. Pressure head distribution above the water table.

In coarse-textured soils in which $K(h)$ has a steep slope and e/K_s is negligible, it can be analytically shown that (Sadeghi et al., 2014):

$$D_{max} = h_e \qquad (2)$$

h_e: pressure head at which $K = e$ (see Fig. 2)

Equation (2) states that the steady-state evaporation rate exhibits a measure for unsaturated conductivity at the pressure head equal to D_{max}. In summary, when $h = D_{max}$, $K = e$.

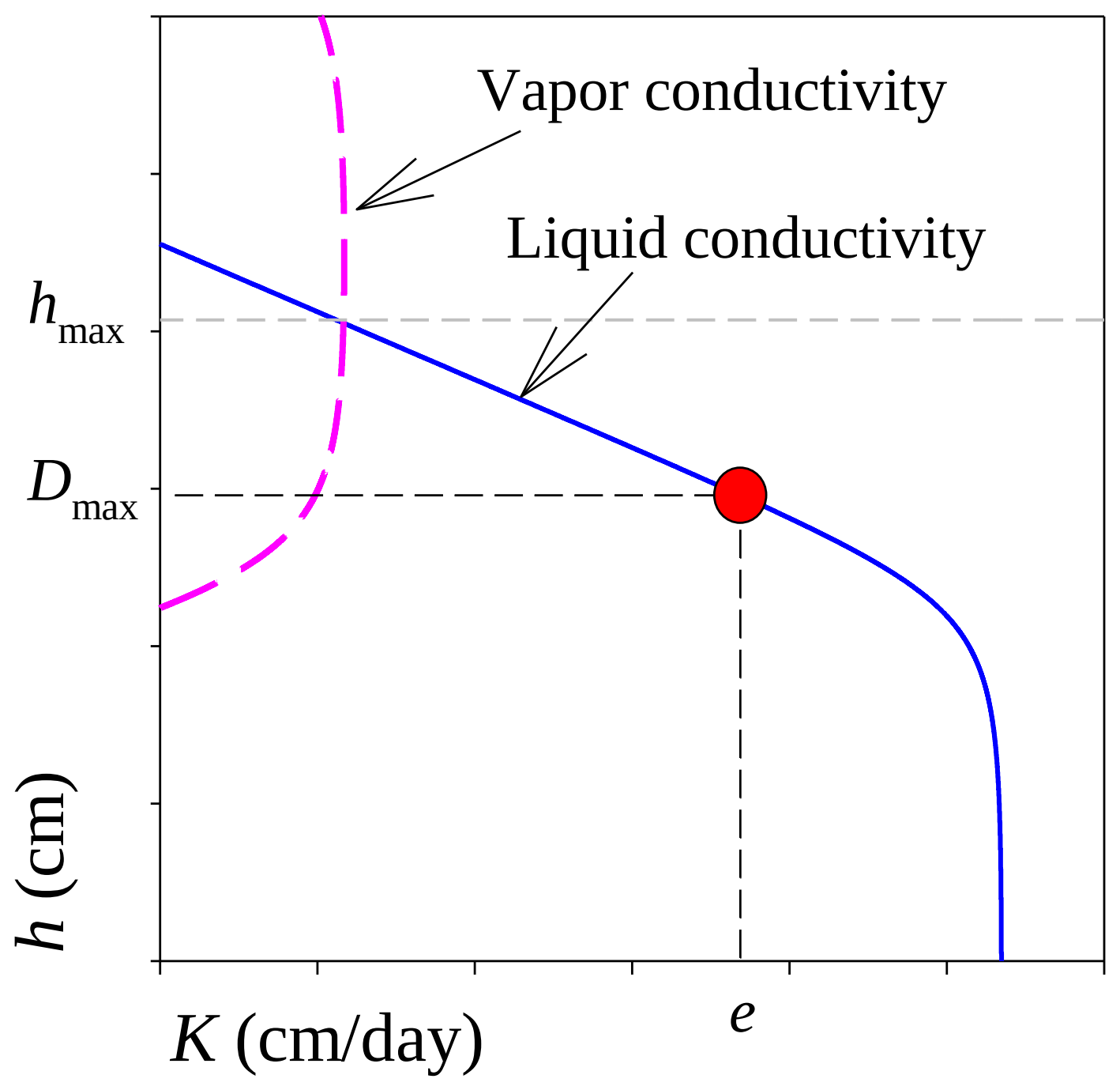


Fig. 2. The new solution to D_{max} from hydraulic conductivity function.

An Analytical Test

An analytical solution for D_{max} using Brooks-Corey function is given by (Sadeghi et al., 2012):

$$D_{max} = h_b \left\{ \frac{\ln(1+e/K_s)}{1+P} - \frac{e}{K_s+e} - (e/K_s)^{-1/P} \left[\frac{2\ln 2}{1-P^2} + \frac{\pi^2/12-\ln 2}{P(1-P)} - 1 \right] \right\} \qquad (3)$$

In coarse-textured soils (P large and e/K_s negligible), D_{max} in Eq. (3) is approximated by $h(K = e)$ from Brooks-Corey conductivity function:

$$D_{max} = h_b (e/K_s)^{-1/P} \qquad (4)$$

Numerical Evaluation

Numerical solutions of Eq. (1) using the van Genuchten (VG), a bimodal, and the Tuller-Or-models validated Eq. (2) for several coarse soils. Fig. 3 shows the results for the VG model.

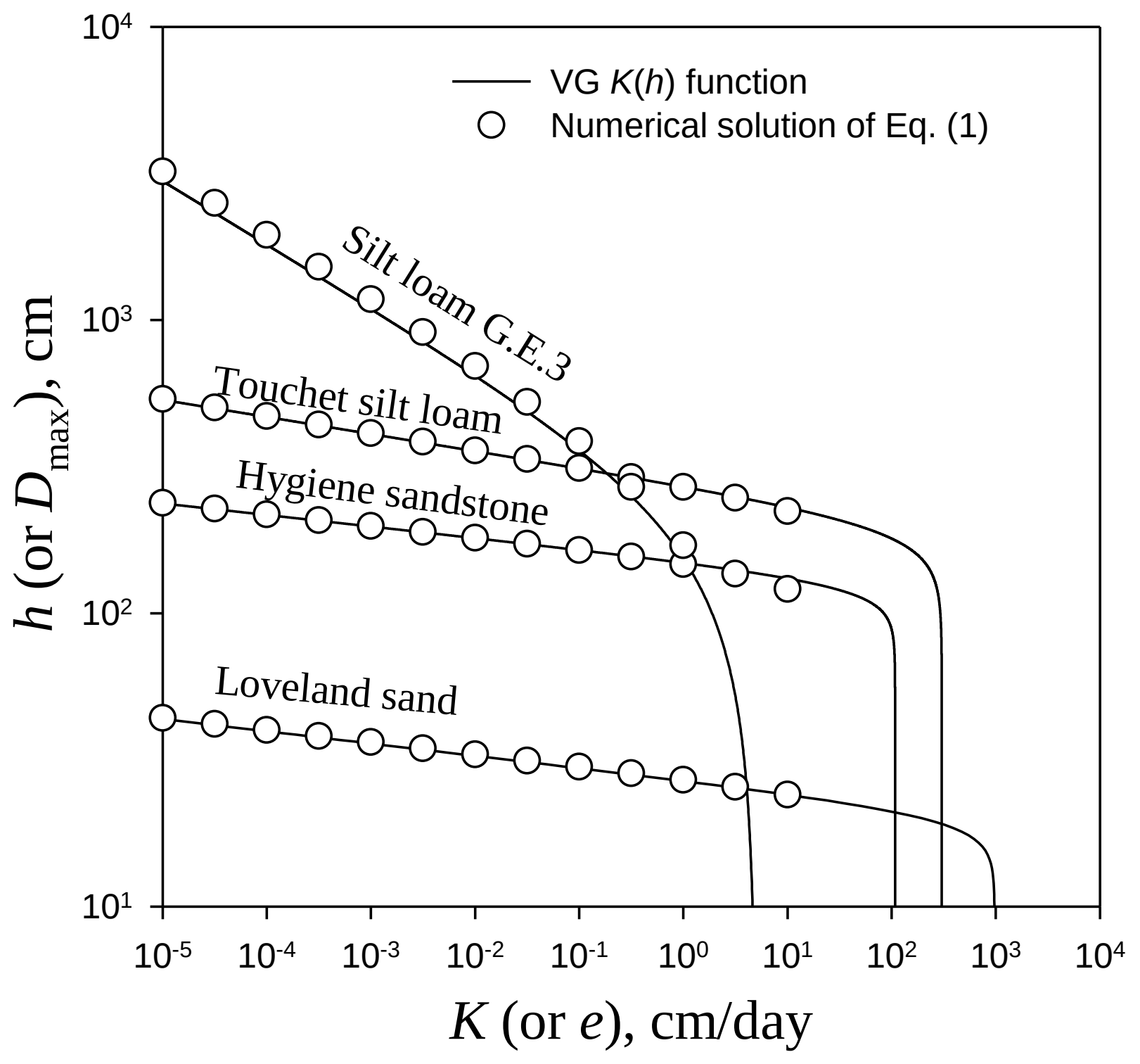


Fig. 3. Numerical solutions of Eq. (1) using the van Genuchten $K(h)$ function.

Experimental Data

Eq. (2) offers a new method to easily determine "column-scale" $K(h)$ in homogeneous soil columns (Fig. 4). It was found that for e > 0.01 cm/day, the water table depth (D) is ≈ D_{max}.

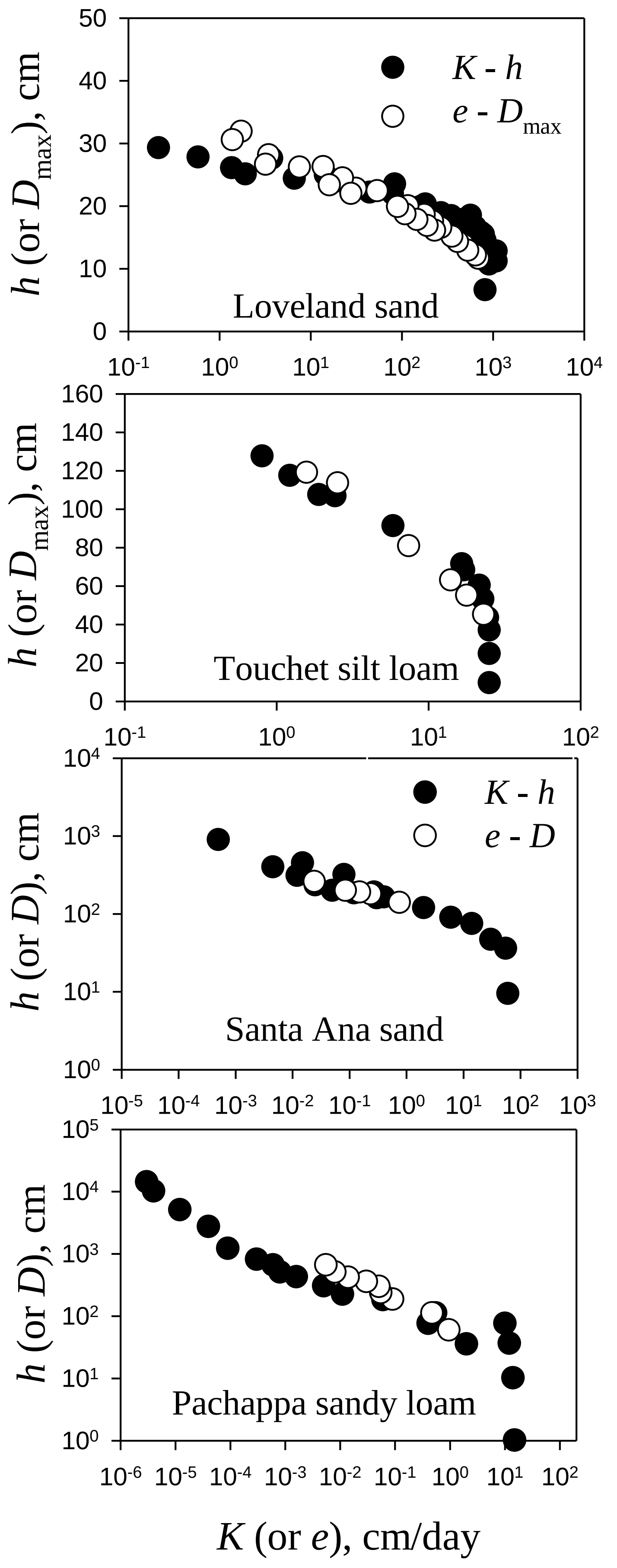


Fig. 4. Experimental data for unsaturated hydraulic conductivity and steady-state evaporation rate.

Effective Unsaturated Conductivity of Layered Soils

Since the steady-state evaporation rate is a macroscopic quantity, it can be considered as an "effective" $K(h)$ for layered soil profiles. In other words, the resultant $K(h)$ values determined with Eq. (2) are representative for the entire D_{max} domain.

In Fig. 5, a 60-cm long soil profile composed of three different sands with a water table at the bottom of the profile was considered. The solution of Eq. (1) for the layered profile was applied as the effective $K(h)$ curve. The agreement between the numerical solutions for the layered system and the introduced equivalent homogeneous profiles indicates the applicability of our proposed approach for deriving effective $K(h)$ of layered soils.

In Fig. 6, we considered a 500-layer periodically repeated binary profile composed of a fine and a coarse sand. A constant water table was considered as the bottom boundary. The effective $K(h)$ obtained using the proposed method lies on the harmonic mean which is consistent with the previous findings, for example, using the homogenization theory (Neuweiler and Eichel, 2006).

Fig. 5. Numerical solution of Eq. (1) for a 3-layer soil profile (thick line) and for its equivalent homogenous soil profile (circles). The former solution (thick line) was considered as the effective (upscaled) conductivity function of the equivalent homogenous profile. Horizontal lines show the layer interfaces.

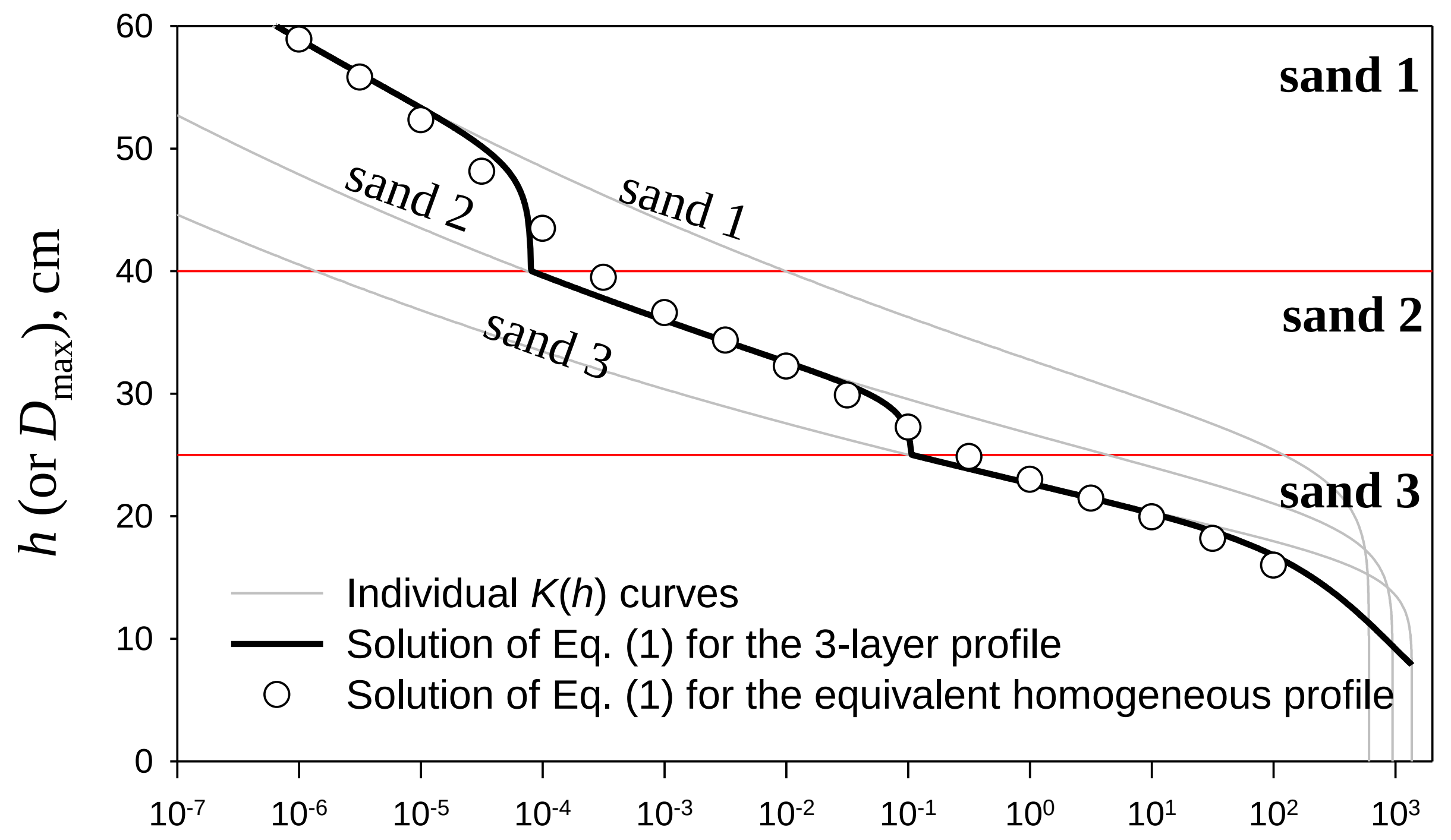
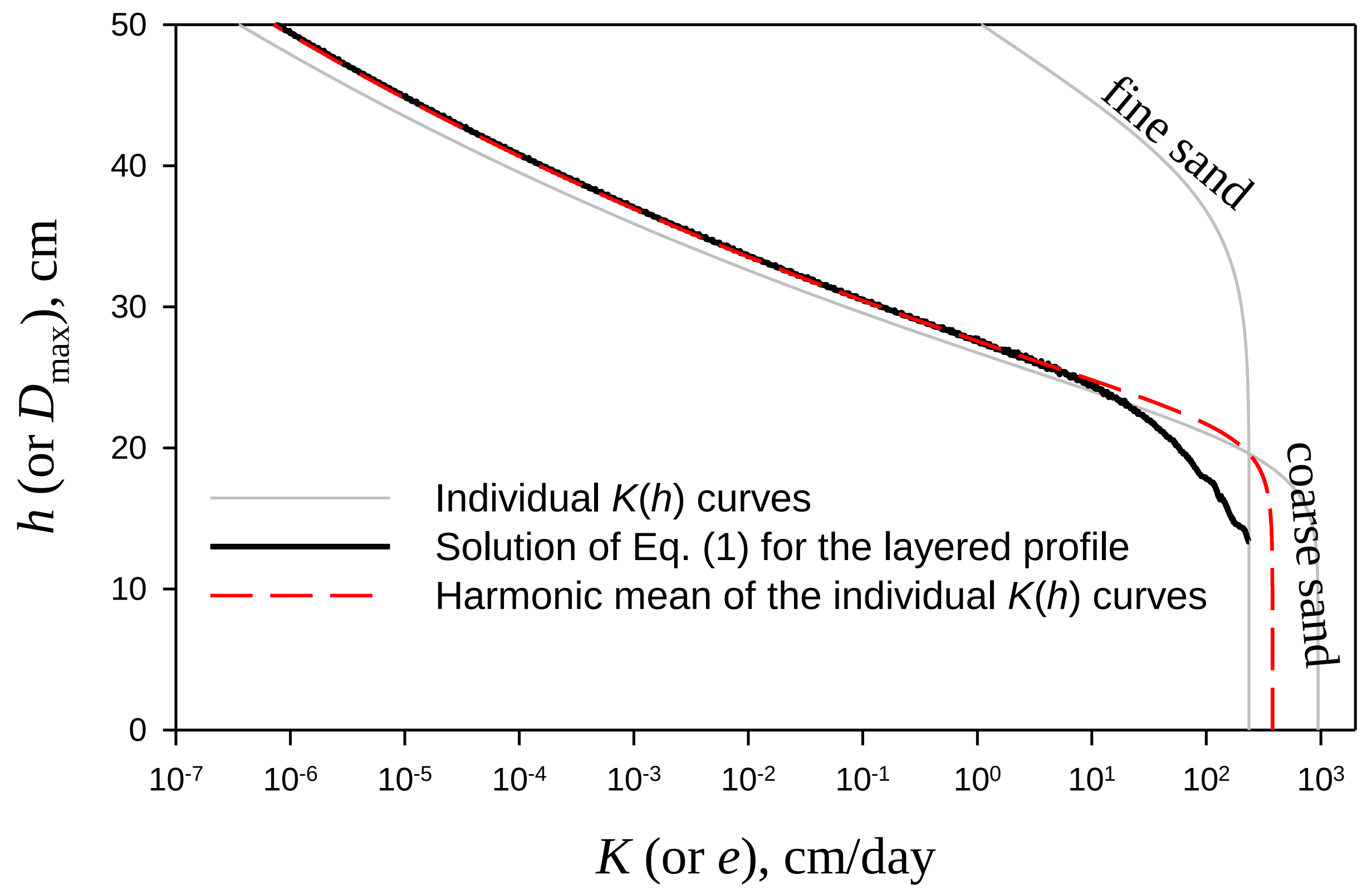


Fig. 6. Numerical solution of Eq. (1) for a 50-cm profile (thick line) composed of 500 periodically repeated layers of a fine sand and a coarse sand as well as for its equivalent homogenous soil profile (circles).



Summary:

Equation (2) indicates that the steady-state evaporation rate from a constant water table is a macroscopic measure of unsaturated hydraulic conductivity in either homogeneous or layered coarse-textured media. Therefore, the presented solution offers an easy method for determination of unsaturated hydraulic conductivity of homogeneous soils and a new approach to effective (upscaled) unsaturated hydraulic conductivity of layered soil systems.

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