Upscaling hydraulic parameters of Green-Ampt model for heterogeneous layered soils

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Introduction

- While Green-Ampt model has been widely used in infiltration calculations through unsaturated soils, upscaling this model for applications in heterogeneous formations remains difficult.
- Main idea of upscaled parameters is to capture infiltration behavior in layered soil formations using only one set of parameters derived from the parameters of individual layers, such that the layered system can be replaced by an equivalent homogeneous medium.
- General p-order power mean was proposed to represent the upscaling schemes of the Green-Ampt model.
- Performance of effective upscaled soil parameters is then evaluated in terms of the ability of the upscaled model in simulating overall behaviors of heterogeneous layered soils.

Materials and methods

- We use repeated binary units which extended from the domain top to the bottom to generate the heterogonous *N* layered soils.
- Green-Ampt model for layered soils is used to calculate infiltration into the heterogeneous layered soils. The infiltration rate f [LT^{-1}], cumulative infiltration F [L] and the travel time of the wetting front t [T] can be expressed:

(1) When the wetting front is in layer 1:

$$f = \frac{K_1}{Z}(Z + H_1) \qquad F = Z(\theta_{s1} - \theta_{01}) = Z\theta_{f1}$$

$$t = \frac{\theta_{f1}}{K_1}Z - \frac{\theta_{f1}H_1}{K_1}ln\left(\frac{Z + H_1}{H_1}\right)$$

(2) When the wetting front is in layer i ($2 \le i \le N$):

$$f = \frac{Z + H_i}{\sum_{j=1}^{i-1}\frac{L_j}{K_j} + \frac{Z - D_{i-1}}{K_i}} \qquad F = \sum_{j=1}^{i-1} L_j\theta_{fj} + (Z - D_{i-1})\theta_{fi}$$

$$t = t_{i-1} + \frac{\theta_{fi}}{K_i}(Z - D_{i-1}) + \theta_{fi}\left[\sum_{j=1}^{i-1} D_j\left(\frac{1}{K_j} - \frac{1}{K_{j+1}}\right) - \frac{H_i}{K_i}\right]ln\left(\frac{Z + H_i}{D_{i-1} + H_i}\right)$$

where K is the saturated hydraulic conductivity [LT^{-1}], H is the suction head at the wetting front [L], Z is the wetting front depth [L], θ_s and θ_0 [L^3L^{-3}] are the saturated and initial water contents respectively, and $\theta_f = \theta_s - \theta_0$ is the initial moisture deficit. L is the thickness of the layer [L], D is the depth from the domain top to the bottom of the layer [L].

- Each individual layer includes the soil parameters of K, H and θ_f . We propose to use the p-order power mean to upscale these parameters.

$$\xi(p) = \left[\frac{1}{N}\sum_{i=1}^{N}\xi_i^p\right]^{1/p}$$

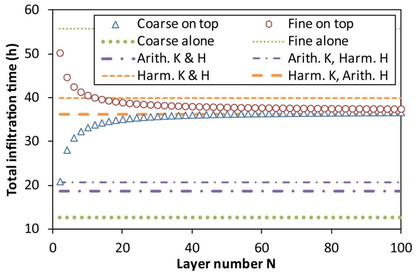
where ξ represents a soil parameter, which can be any of K, H or θ_f . Note that the arithmetic (p = 1), geometric (p = 0), and harmonic (p = -1) means are all special cases of the p-order power mean.

Upscaling hydraulic parameters

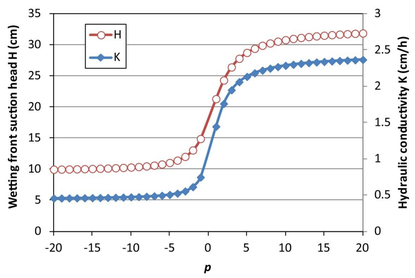
- The θ_f is upscaled using the arithmetic mean to make the final cumulative infiltrations in the equivalent homogeneous medium and the layered formation equal. The remaining parameters of K and H are then upscaled using arithmetic mean, harmonic mean or more general p-order power mean, which translate into 8 potential combinations of upscaling schemes.

The results of whether optimal p values can be obtained in different schemes.

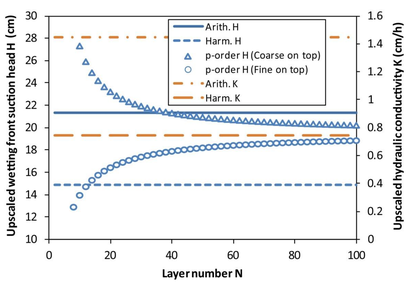
Type of binary unit		Arithmetic K	Harmonic K	p-order power K
Coarse on top	Arithmetic H	-	-	**No**
	Harmonic H	-	-	**No**
	p-order power H	**No**	**Yes**	*N/A*
Fine on top	Arithmetic H	-	-	**No**
	Harmonic H	-	-	**No**
	p-order power H	**No**	**Yes**	*N/A*



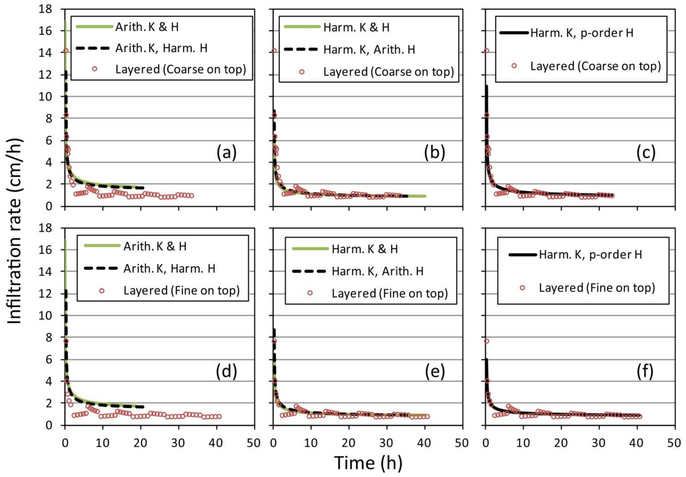
The total infiltration time as functions of the layer number. The times for the homogeneous medium by simple upscaling schemes are also shown.



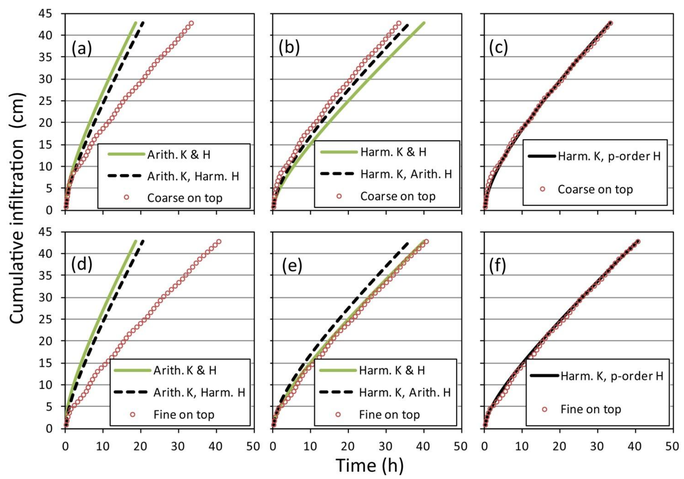
The p-order power mean values of hydraulic conductivity and wetting front suction head as functions of p value.



Upscaled soil parameters as functions of layer number for each scheme.



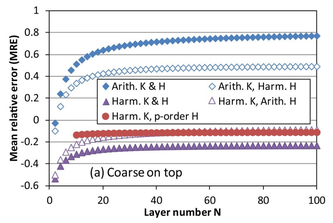
Infiltration rate curves versus time for each scheme when layer number N is 10 for the layered soil formations.

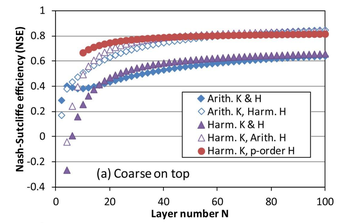


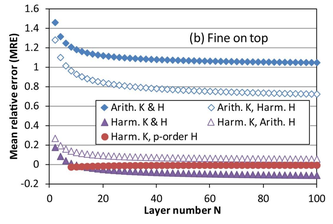
Cumulative infiltration curves versus time for each scheme when layer number N is 10 for the layered soil formations.

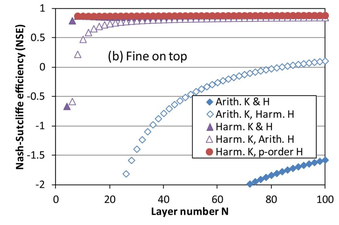
Evaluation of performance

- Upscaling schemes were based on infiltration time, the evaluations are based on infiltration rate so the determination and evaluation stages are independent.
- MRE describes the difference of infiltration rate between the equivalent homogeneous medium and the layered formation, while the NSE represents the matching degree between them.









Mean relative errors (MRE) of infiltration rates between heterogeneous layered soils and equivalent homogeneous medium for each scheme.

Nash-Sutcliffe efficiency (NSE) values of infiltration rates between heterogeneous layered soils and equivalent homogeneous medium for each scheme.

Conclusions

- Only the schemes using harmonic mean for K could lead to reasonable optimal p values for H, which is supported by the smaller mean relative error (MRE) and the higher Nash-Sutcliffe efficiency (NSE) for the infiltration rates.
- Structure of the layered formations influences the upscaling results significantly. When the layer number becomes large, the total infiltration times for these two formation structures converge to the same value, indicating that the layered formations tend to show homogeneity and the layer structure becomes less important.
- Upscaled Green-Ampt parameters using harmonic mean for K and p-order power mean for H in the equivalent homogenous medium can capture the infiltration rate and cumulative infiltration of heterogeneous layered soils.

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