Improved Surface Area Estimation Based on Surface Curvedness

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Introduction

- **Accurate determination of interfacial/surface areas in multiphase systems is of essence to enhance our understanding of multiphase flow and mass transfer processes in porous media. X-ray micro-CT provides promising means to estimate surface areas from three-dimensional segmented images (Fig.1).**
- **Several classes of estimators have been proposed in literature, including methods that assign weights to specific voxel configurations, that reconstruct approximate surfaces with polygons or integrate surface voxels intersecting with a set of uniformly distributed lines. Especially, weighted-voxel based methods that determine weights by optimizing planes that approximate curved surfaces yield unsatisfactory estimates for planar surfaces when compared to simple surface-voxel-face counts.**
- **To overcome this limitation, we present a new surface area (SA) estimator that is based on surface curvedness computed from principal curvatures. A curvedness threshold is applied to discern surface voxels with either curved or planar surface neighborhoods. While for voxels with curved neighborhoods a weighted-voxel method is applied, voxels with planar neighborhoods are treated with the surface-voxel-face count method.**

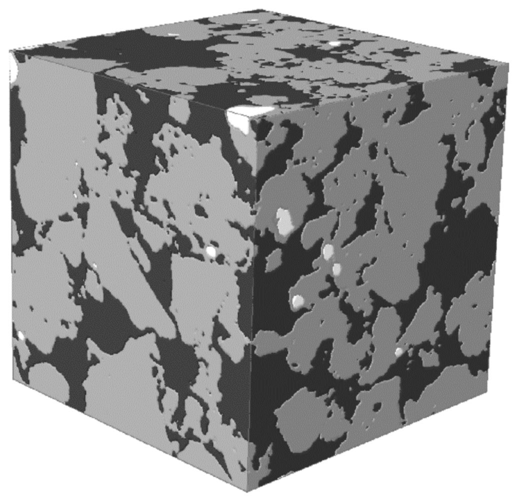

Figure 1: Rendering of segmented CT data of a Brazilian Oxisol. The solid-air interface exhibits a combination of planar and curved regions. Adapted from Vaz et al. (2015).

Weighted Voxel Surface Area Estimator - Lindblad

- **Weighted SA estimators are designed for discrete binary three-dimensional data representing object voxels and background voxels. They assign weights to different combinations of voxel configurations in either 2×2×2 or 3×3×3 neighborhoods, and determine the total surface area of an object by summing the weights of all voxels that are part of it.**
- ***Lindblad (2005)* developed a method that determines optimal weights for all possible voxel configurations in a 2×2×2 neighborhood via a Monte Carlo optimization scheme that minimizes the variance of randomly oriented digitized planar surfaces.**
- **An m-cube (short for Marching Cube) is the cube bounded by the centers of eight voxels of a 2×2×2 neighborhood. Each corner of the m-cube corresponds to a voxel center. A m-cube can be seen as the dual of the vertex that is shared by its eight surrounding voxels. Correspondingly, each voxel is shared by its eight surrounding m-cubes. In a binary image, the number of possible configurations of the eight voxels is 2^8 = 256. Because of symmetry, the 256 configurations can be grouped into 14 m-cubes cases each assigned a different weight for SA calculation (Fig. 2).**

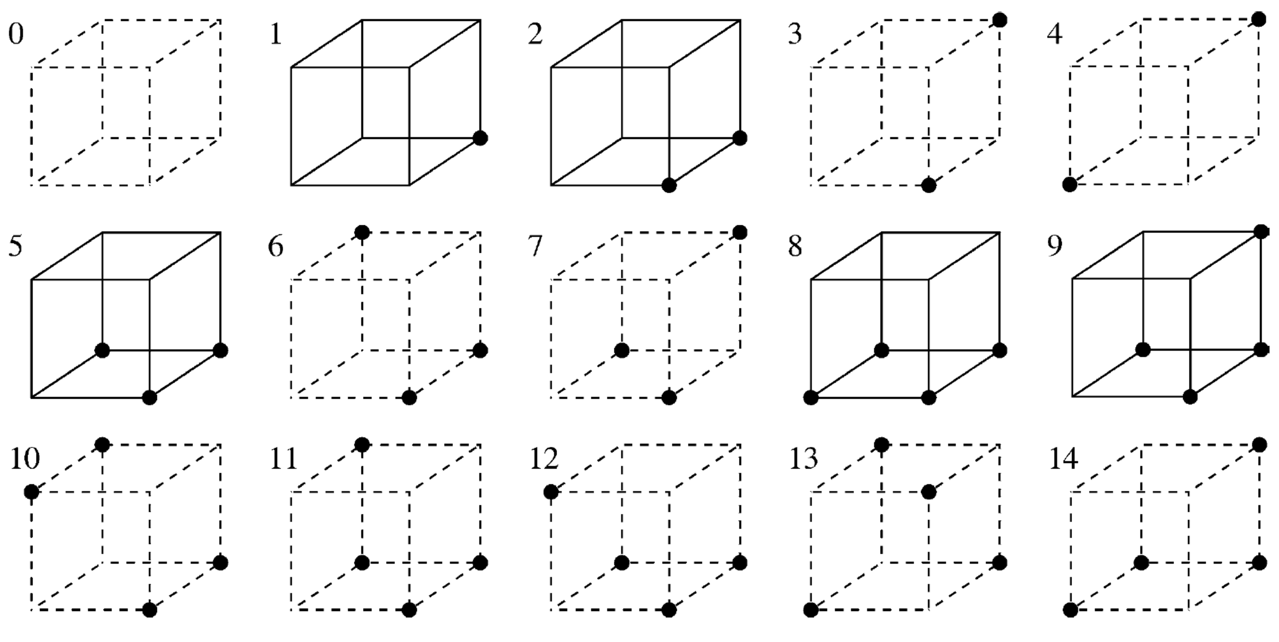

Figure 2: m-Cubes of 2×2×2 voxels. Voxel centers denoted by a • are inside the object. Only Cases 1, 2, 5, 8, and 9 are for planar surfaces. Adapted from *Lindblad (2005)*.

- **Weighted surface area estimators such as proposed by** Lindblad (2005) **commonly yield higher relative errors for planar surfaces than simple surface-voxel-face count (SVFC) methods.**

Principal Curvature and Curvature Index

- **We propose a combination method that utilizes the** *Lindblad (2005)* **approach for curved regions and the SVFC method for planar regions and apply a curvature index to discern voxels with either planar or curved neighborhoods.**
- **The two principal curvatures (k_1, k_2) at a point on a surface are the eigenvalues of the shape operator at that point and provide a measure of how the surface bends. Gaussian curvature (K=$k_1 \times k_2$) is defined as the product of the principal curvatures and mean curvature (S=[k_1+k_2]/2) as their mean value.**
- **We can compute** K **and** S **from the intrinsic equation of a surface object** (*Thirion and Gourdon, 1993*):

$$S = \frac{1}{2h^{3/2}}\left[f_x^2(f_{yy}+f_{zz}) - 2f_y f_z f_{yz} + f_y^2(f_{xx}+f_{zz}) - 2f_x f_z f_{xz} + f_z^2(f_{xx}+f_{yy}) - 2f_x f_y f_{xy}\right]$$

$$K = \frac{1}{h^2}\begin{bmatrix} f_x^2(f_{yy}f_{zz} - f_{yz}^2) + 2f_y f_z(f_{xz}f_{xy} - f_{xx}f_{yz}) \\ + f_y^2(f_{xx}f_{zz} - f_{xz}^2) + 2f_x f_z(f_{yz}f_{xy} - f_{yy}f_{xz}) \\ + f_z^2(f_{xx}f_{yy} - f_{xy}^2) + 2f_x f_y(f_{xz}f_{yz} - f_{zz}f_{xy}) \end{bmatrix}$$

$$\Delta = S^2 - K$$

$$k_1 = S + \sqrt{\Delta}$$

$$k_2 = S - \sqrt{\Delta}$$

- **Curvature Index** (*CI*) **is defined as the magnitude of the vector formed by the principal curvatures in a (k_1, k_2) parameter plane** (*Koenderink and van Doorn, 1992*):

$$CI = \sqrt{\frac{k_2^2 + k_1^2}{2}}$$

Preliminary Results – New Combination Method

- **Based on the definition of the Curvature Index (*CI*), for a voxel in a planar neighborhood *CI*=0 and for a voxel in a curved region *CI*≠0.**
- **The proposed new combination method computes *CI* for every surface voxel and applies the weights proposed by Lindblad for voxels with *CI*≠0 and the entire voxel surface that is part of the interface for voxels with *CI*=0.**
- **For a preliminary proof of concept we chose four basic geometries for which the surface areas can be calculated analytically and compare the original *Lindblad (2005)* approach, the simple surface-voxel-face count (SVFC) method, and the proposed new combination method based on the relative error ε that is computed as:**

$$\varepsilon = \frac{Estimated\ SA - Analytical\ SA}{Analytical\ SA}$$

- **To illustrate how the CT scan resolution potentially affects the SA estimates, ε is computed and plotted for various spatial object resolutions.**

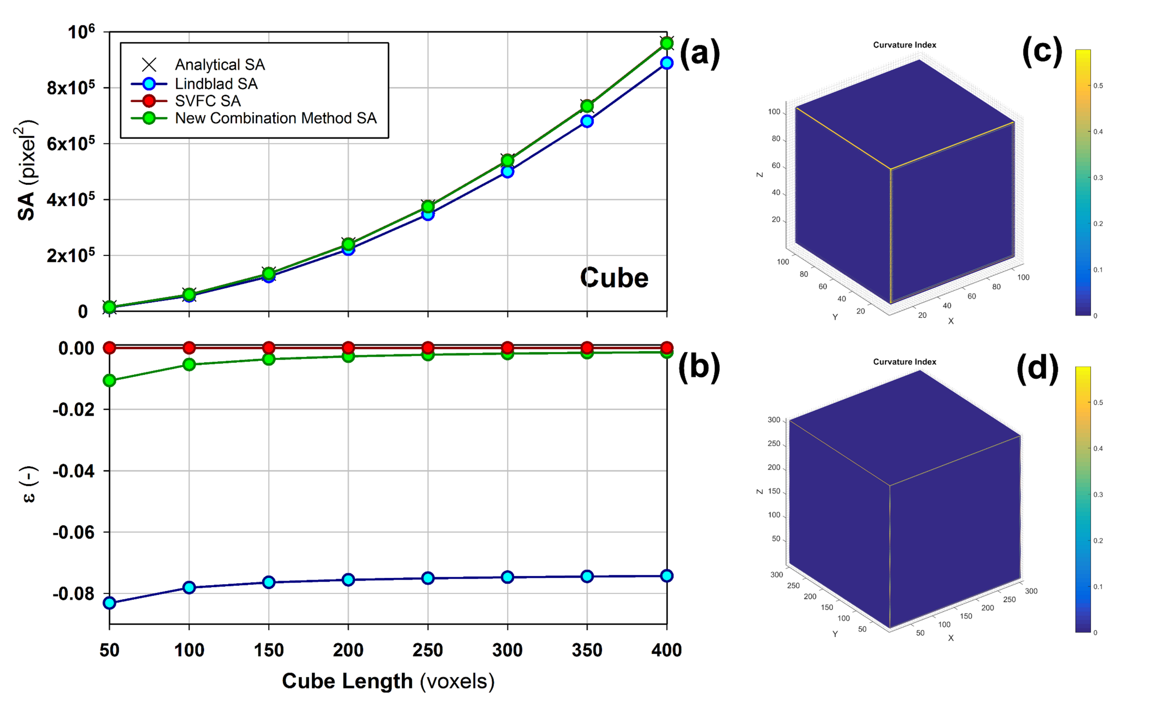

Figure 3: Comparison of the original Lindblad (2005) approach, the simple surface-voxel-face count (SVFC) method, and the proposed new combination method. Surface area as a function of object resolution (a); relative error as a function of object resolution (b); curvature indices for cubes at 100×100×100 (c) and 300×300×300 (d) resolutions.

Preliminary Results - Continued

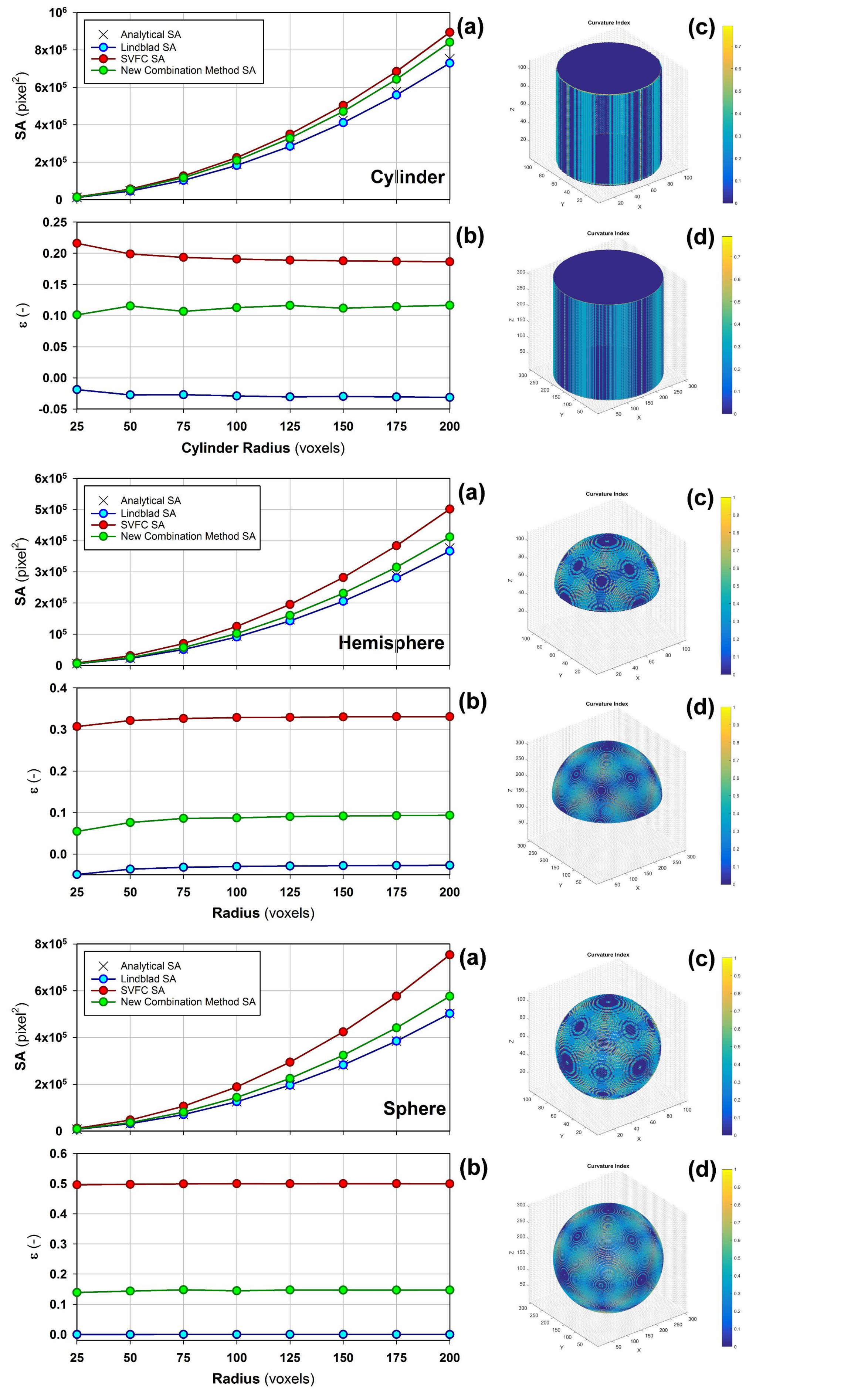

Figure 4: Comparison of the original Lindblad (2005) approach, the simple surface-voxel-face count (SVFC) method, and the proposed new combination method for various geometries. Surface area as a function of resolution (a); relative error as a function of resolution (b); and curvature indices for objects at different resolutions (c, d).

Conclusions and Future Work

- **The proposed new combination method improves surface area estimates for objects with extended flat regions.**
- **However, the Lindblad (2005) approach still performs better for objects with extended curved surfaces. This leads to the conclusion that the curvature index should be determined for a larger region around a surface voxel rather than for each individual voxel.**
- **A potential approach to improve the proposed combination method is to apply the partial derivative of the *CI* to discern if a surface voxel belongs to a flat or curved region. This is part of ongoing efforts.**

References

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