Bayesian data-worth analysis for unsaturated soil hydraulic parameter estimation

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Introduction

- It is widely recognized that the movement of water and heat is closely coupled and they affects each other. Thus reliable estimation of soil hydraulic and thermal parameters is essential for predicting water movement in unsaturated soil.
- Soil water content and temperature are commonly used measurements for the inverse modeling of hydraulic and thermal properties. Nevertheless, there is still a lack of systematic investigation about the data worth of these two types of measurements in characterizing unknown soil hydraulic and thermal parameters.

System Model

Governing equation:

$$\begin{cases} \dfrac{\partial \theta}{\partial t} = \dfrac{\partial}{\partial x_i}\left[K(h)\dfrac{\partial h}{\partial x_i}\right] + \dfrac{\partial}{\partial x_j}\left[K(h)\dfrac{\partial h}{\partial x_j} + K(h)\right] & \text{Water flow} \\ C(\theta)\dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial x_j}\left[\lambda_{ij}(\theta)\dfrac{\partial T}{\partial x_i}\right] - C_w q_j \dfrac{\partial T}{\partial x_j} & \text{Heat transport} \end{cases}$$

$K(h)$ is the hydraulic conductivity function, which is described by van Genuchten-Mualem (VGM) model,

$$K(h) = K_s S_e^{0.5}\left[1-(1-S_e^{1/m})^m\right]^2, \quad m = 1-1/n$$

$$S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r} = \begin{cases} \dfrac{1}{(1+|\alpha h|^n)^{1-1/n}}, & h<0 \\ 1, & h \geq 0 \end{cases}$$

$\lambda_{ij}(\theta)$ is the apparent thermal conductivity given by

$$\lambda_{ij}(\theta) = \lambda_T C_w |q| \delta_{ij} + (\lambda_L - \lambda_T) C_w \frac{q_i q_j}{|q|} + \lambda_0(\theta)\delta_{ij}$$

$$\lambda_0(\theta) = b_1 + b_2\theta + b_3\theta^{0.5}$$

Methods

➢ Bayesian inference

The posterior distribution of model parameters **m** can be obtained by assimilating the measurements **d** according to Bayes' theorem

$$p(\mathbf{m}\,|\,\mathbf{d}) = \frac{p(\mathbf{m})p(\mathbf{d}\,|\,\mathbf{m})}{p(\mathbf{d})} \propto p(\mathbf{m})L(\mathbf{m}\,|\,\mathbf{d})$$

In this study, we resorted to sampling the posterior distribution by MCMC.

➢ Information metric

The expected utility rooted in relative entropy is used to quantify the information content of measurements

$$u(\mathbf{d}) = \int_{\Theta} p(\mathbf{m}\,|\,\mathbf{d}) \ln\left[\frac{p(\mathbf{m}\,|\,\mathbf{d})}{p(\mathbf{m})}\right] d\mathbf{m}$$

Note that the actual measurements are unavailable in the computational process. Thus the expected utility value is calculated by averaging on all possible measurement realizations.

Case Study

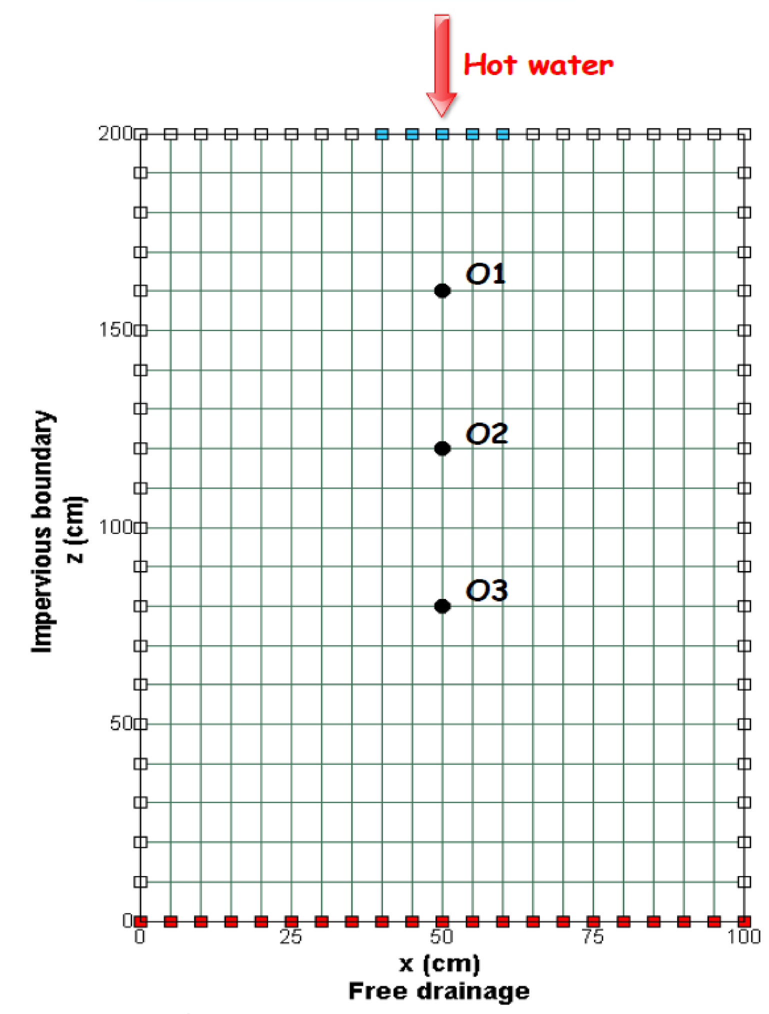

Figure 1. The study domain, boundary conditions and three observation locations.

Case description:

HYDRUS-2D was employed to simulate the process of hot water infiltrating into soil profile through a single infiltration ring.

The whole simulation time lasted for 10 hours and measurements were obtained at 3 locations with a 1-hour interval.

Table 1. The upper and lower bounds of parameters of interest.

Parameters	α (cm^{-1})	n (−)	K_s ($cm\ h^{-1}$)	b_1 ($\times 10^{12}$ kg cm h^{-3} K^{-1})	b_2 ($\times 10^{12}$ kg cm h^{-3} K^{-1})	b_3 ($\times 10^{12}$ kg cm h^{-3} K^{-1})
Lower bound	0.019	1.36	4.828	2.1788	0.4778	2.1742
Upper bound	0.093	2.37	11.404	4.8569	2.4261	5.1835

Results and Discussion

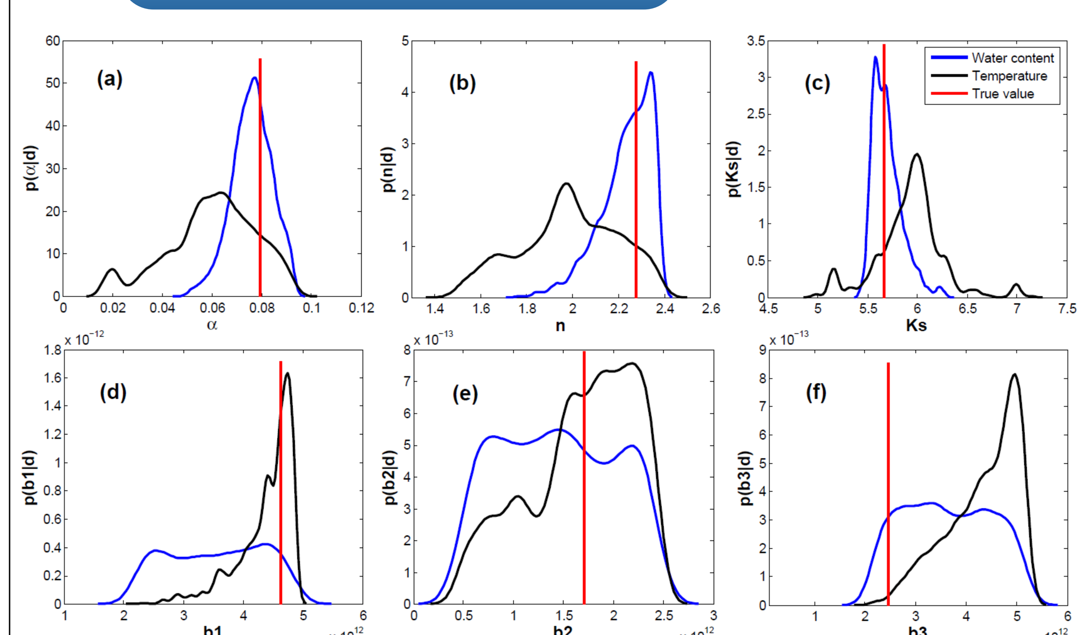

Figure 2. Posterior probability distributions of estimated parameters.

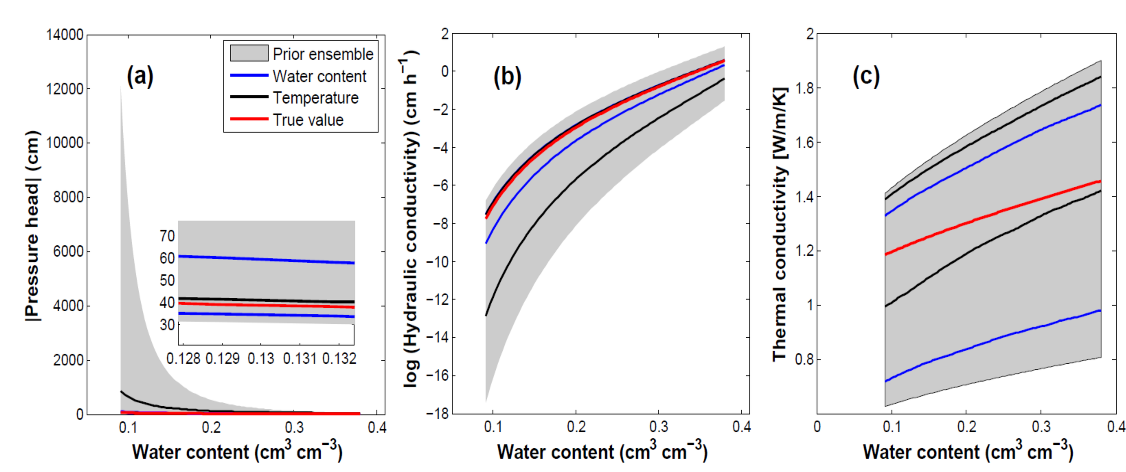

Figure 3. The true and estimated (a) water retention curves, (b) hydraulic conductivity functions and (c) thermal conductivity functions. The lines with the same color stand for the upper and lower bounds of uncertainty (95% confidence intervals).

The results shown in Figures (2, 3) were obtained by assimilating water content and temperature measurements with error levels of $\sigma_W = 0.01\ cm^3\ cm^{-3}$, $\sigma_T = 1.0$ °C.

Table 2. Data-worth values of different types of measurements with five measurement error levels.

Standard deviations of measurement error (W: $cm^3\ cm^{-3}$; T: °C)	W	T
$\sigma_W = 0.01$, $\sigma_T = 0.2$	6.180(0.010)	10.051(0.005)
$\sigma_W = 0.02$, $\sigma_T = 0.4$	4.328(0.008)	8.711(0.010)
$\sigma_W = 0.03$, $\sigma_T = 0.6$	3.441(0.010)	7.414(0.008)
$\sigma_W = 0.04$, $\sigma_T = 0.8$	2.902(0.007)	6.436(0.013)
$\sigma_W = 0.05$, $\sigma_T = 1.0$	2.541(0.007)	5.718(0.008)

Table 3. Data-worth values of measurements sampled from different observation locations with typical in-situ measurement error levels of $\sigma_W = 0.03\ cm^3\ cm^{-3}$, $\sigma_T = 1.0$ °C.

Observed location	W	T	WT
O1	1.308(0.005)	3.454(0.008)	3.802(0.005)
O2	2.135(0.008)	3.938(0.007)	4.499(0.005)
O3	2.594(0.004)	3.121(0.006)	4.104(0.005)
O1O2	2.603(0.006)	5.010(0.013)	5.605(0.009)
O1O3	3.024(0.004)	4.716(0.010)	5.655(0.009)
O2O3	3.093(0.007)	4.834(0.007)	5.801(0.008)
O1O2O3	3.441(0.010)	5.718(0.008)	6.594(0.010)

Note: W: water content; T: temperature; WT: water content + temperature.

Conclusions

✓ The information content of the measurement decreased as the measurement error level increased;

✓ With the typical in-situ measurement error level, the temperature data were more informative than the water content measurements, and jointly assimilating these two types of measurements provided non-redundant information.